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Optimal climate policies in a dynamic multi-country equilibrium model ☆

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Abstract

This paper develops a dynamic general equilibrium model with an arbitrary number of different regions to study the economic consequences of climate change under alternative climate policies. Regions differ with respect to their state of economic development, factor endowments, and climate damages and trade on global markets for capital, output, and exhaustible resources. Our main result derives an optimal climate policy consisting of an emissions tax and a transfer policy. The optimal tax can be determined explicitly in our framework and is independent of any weights attached to the interests of different countries. Such weights only determine optimal transfers which distribute tax revenues across countries. We infer that the real political issue is not the tax policy required to reduce global warming but rather how the burden of climate change should be shared via transfer payments between different countries. We propose a simple transfer policy which induces a Pareto improvement relative to the Laissez faire solution. A calibrated example quantifies Pareto-improving transfers between rich and poor countries.

JEL classification: E61; H21; H23; Q43; Q54

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0. Introduction

On December 12, 2015, 195 countries joined the Paris Agreement to strengthen the global response to climate change. Its central aim is to keep the increase in temperature relative to pre-industrial level below two degrees Celsius until the end of the century. Individual commitments how to achieve this goal, however, are non-binding and involve voluntary climate policies chosen on the national level, so called 'Nationally Determined Contributions'. This outcome reflects the complexity of any negotiations about joint climate policies which are shaped by the bargaining power of individual countries and the trade-offs between their political interests.

These observations suggest that climate change is inherently an *economic problem* and understanding the incentives for individual regions to implement a given climate policy is key for the success of any climate agreement. Conceptually, this calls for a theoretical framework which incorporates the trade-off between the interests of different regions and permits to analyze the effects of alternative climate policies at the regional level. In this paper, we develop such a multi-region model and derive an optimal climate policy that each region has an incentive to implement.

To successfully combat climate change, each country must contribute to the common goal by reducing its emissions. A major obstacle to determine these individual contributions is regional heterogeneity. For instance, regions differ considerably in their dependence on fossil fuels and the mix of technologies they use to produce energy inputs. Thus, reducing emissions is more costly for some regions than for others. In addition, various other differences such as state of economic development, future growth prospects, or vulnerability to climate damages play an important role in climate negotiations. This raises a first question that we will address in this paper: *How much should each region contribute to the common objective of reducing emissions?*

In market economies, any equilibrium allocation is the outcome of decentralized decisions made by firms and consumers who respond to incentives and taxes set by governments. One way to reduce emissions is to levy a tax on emissions. Thus, a second questions to be addressed is: *Which tax policy should each region adopt to achieve its desired reduction in emissions*?

For a climate agreement to be successful, each region must have an incentive to implement the proposed policy. Thus, a third question to be answered is: *How can climate policies be designed such that each region has an incentive to join the climate agreement?*

The present paper addresses these questions in a multi-region framework which incorporates several sources of regional heterogeneity which play a crucial role in climate negotiations. Our model builds on the single-region framework in Golosov et al. (2014) to which we add several new dimensions. First, we adopt a multi-region structure featuring an arbitrary number of different regions. This is clearly required for our analysis. Second, we devise a different model of the production process which distinguishing explicitly the *resource stage* at which fossil fuels are *extracted* and the *energy stage* at which they are *used* to produce energy such as electricity or heat. This distinction is important because empirically regions differ considerably not only in their dependence on fossil fuels but also in the mix of technologies to produce energy outputs. Modelling the energy stage explicitly allows us to capture these differences and study how climate policies induce transitions from dirty to clean technologies in each region. Finally, we allow for climate damages to differ across regions. This is another important source of heterogeneity,

notably because poorer countries tend to be more vulnerable to climate change. The economic part is complemented by a climate model describing how emissions evolve in the atmosphere and damage the economy.

With these features, our model falls into the class of integrated assessment models which incorporate the full interactions between climate variables and the economic production process. In the literature, a large class of these models is based on the DICE framework pioneered by Nordhaus (1977) and its multi-region extension, the RICE model developed in Nordhaus and Yang (1996) and refined in Nordhaus and Boyer (2000). A typical feature of these models stressed in Hassler et al. (2016) is that solutions are derived as planning problems without explicit market structures and prices. Thus, these models make only limited use of dynamic general equilibrium theory which confines the class of policies that can be analyzed. In addition, the RICE-framework in conjunction with the employed methodology entails strong restrictions on trade between regions.

The model developed in Golosov et al. (2014) takes full advantage of dynamic general equilibrium theory with explicitly defined markets and price formation.[1] Hassler and Krusell (2012) provide an extension to a multi-region framework which distinguishes oil-producing and oil-consuming countries. To preserve analytical tractability, they impose strong restrictions on trade between regions which are only allowed to trade oil which is the only fossil fuel.[2] A major difference of our model to Hassler and Krusell (2012) is that we allow for trade between regions and intertemporal borrowing and lending on an international capital market. In fact, this assumption will be key for our results.

The general contribution of our analysis to the literature are the answers to the questions posed above. First, we show that there is a unique efficient allocation which determines the optimal level of emissions for each region. Second, we show that this efficient allocation can be implemented by a uniform tax on emissions for which we derive a closed form solution. Third, we devise a transfer scheme which distributes the revenue from taxation such that each region has an incentive to implement the optimal emissions tax.

A crucial feature of our model to obtain these results is that the *efficiency problem* of determining an optimal emissions tax can strictly be separated from the *distributional issue* how global tax revenue should be shared via transfers. This separability result requires a standard restriction on consumer preferences combined with borrowing and lending between regions on a frictionless capital market. Static versions of this result first proved by Bergstrom and Cornes (1983) are well-known in the public goods literature. To the best of our knowledge, we are the first to extend it to a dynamic setting and apply it to the climate problem. This is the methodological contribution of our paper.

The bargaining power of regions in our model is represented by a weighting scheme which aggregates welfare in each region to a single utility index. An important consequence of separability is that the optimal emissions tax does not depend on this aggregation. Thus, determining the optimal carbon tax involves no trade-off between political interests and regions could directly agree on this policy. Based on this finding, our analysis suggests that the major political issue is

[1] Recent models based on the same paradigm are Barrage (2017), Gerlagh and Liski (2016, 2018), or Rezai and van der Ploeg (2015, 2016) who extend the model in Golosov et al. (2014) in various directions.

[2] A related class of models studies climate policies in a two-region setting, e.g., Daubanes and Grimaud (2010), Bretschger and Suphaphiphat (2014), or van den Bijgaart (2017). The model from Hassler and Krusell (2012) is further extended in Hassler et al. (2017) to include directed technical change.

how tax revenue should be shared via transfer payments. Only the choice of a transfer scheme should therefore be the subject of negotiations.

To determine transfers in accordance with incentive constraints, our final theoretical result proposes a simple transfer policy under which each region is strictly better off relative to the Laissez faire scenario where no measures against climate change are taken. The proposed policy thus satisfies the property of *individual rationality* discussed, e.g., in Eyckmans and Tulkens (2003) which seems a minimal requirement for the optimal tax policy to be implemented by each region. We present a calibrated numerical example to quantify the range and size of Pareto-improving transfers between rich and poor countries. The results show that rich countries can afford to transfer initially 1.6% and subsequently up to 2.3% of their GDP to poor countries and would still benefit from a global agreement to implement the optimal tax policy.

The paper is organized as follows. Section 1 introduces the model. The decentralized equilibrium solution under different climate policies is studied in Section 2. Section 3 studies optimal allocations obtained as solutions to a planning problem. Optimal climate policies which implement the optimal solution as an equilibrium allocation are studied in Section 4. A calibrated example presented in Section 5 quantifies our results. Extensions of our basic framework and robustness of our results are discussed in Section 6. Section 7 concludes, mathematical proofs are placed in the appendix.

1. The Model

1.1. World economy

The world economy is divided into $L \geq 1$ regions, indexed by $\ell \in \mathbb{L} := \{1, \ldots, L\}$. Each region $\ell \in \mathbb{L}$ pursues its own interests and takes autonomous political decisions. Although each region typically represent unions or groups of different countries, we will nevertheless refer to region ℓ as a country. Regions are geographically or institutionally separated, which imposes certain restrictions on trade between them.

The production process in each region $\ell \in \mathbb{L}$ decomposes into three stages. The *final sector* produces a consumable output commodity based on a set of inputs including energy goods and services. The second stage consists of a collection of *energy sectors* which produce these goods and services based either on renewable or exhaustible resources. The third stage is represented by the *resource sectors* which extract the domestic stock of exhaustible resources.

These different *production stages* together with a global *climate model* and a description of the *consumption sector* in each region constitute the main building blocks of our model which will be described in detail in the following sections.

1.2. Production sectors

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. There are $I + 1$ production sectors in each region $\ell \in \mathbb{L}$ which are identified by the index $i \in \mathbb{I}_0 := \{0, 1, \ldots, I\}$. Sector $i = 0$ is the *final sector* while $i \in \mathbb{I} := \{1, \ldots, I\}$ identifies the different *energy sectors*. Each sector $i \in \mathbb{I}_0$ consist of a single representative firm which employs labor $N^\ell_{i,t} \geq 0$ and capital $K^\ell_{i,t} \geq 0$ as production factors in period t.

Final sector

Sector $i = 0$ in region $\ell \in \mathbb{L}$ produces final output in period t using the technology

$$Y^\ell_t = (1 - D^\ell_t) Q^\ell_{0,t} F_0(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}). \tag{1}$$

Here, $(E^{\ell}_{i,t})_{i\in\mathbb{I}}$ is as collection of energy inputs used in addition to labor and capital in production. The term $Q^{\ell}_{0,t} > 0$ in (1) is an exogenous, possibly time- and region-specific productivity parameter which is diminished by damages due to climate change. The latter is measured by a damage index $D^{\ell}_t \in [0, 1[$ which will be a function of total CO_2 concentration in the atmosphere to be specified below.

Energy sectors

Energy is supplied by sectors $i \in \mathbb{I}$. Their outputs should be broadly interpreted as energy goods like electricity and heat or services like transportation. We distinguish 'exhaustible' and 'renewable' energy sectors depending on whether they base their production on an exhaustible resource like coal, oil, and natural gas or a renewable resource like wind, water, and solar energy.

Let $\mathbb{I}_x \subset \mathbb{I}$ denote the subset of exhaustible energy sectors. Each such sector $i \in \mathbb{I}_x$ is uniquely identified by the underlying resource on which production is based (like 'coal' used for 'coal-fired power generation' or 'oil' used to provide 'fuel-based transportation services'). The amount of exhaustible resource $i \in \mathbb{I}_x$ used in region $\ell \in \mathbb{L}$ at time t is denoted by $X^{\ell}_{i,t} \geq 0$. Exhaustible resources are typically an essential input to production in the respective sector and generate emissions proportional to their usage in production. Energy sectors thus represent the production stage at which emissions are potentially generated. Sectors which employ renewable sources do not cause emissions.[3]

With the previous distinction, the technology used by an *exhaustible energy sector* $i \in \mathbb{I}_x$ to produce energy output in period t takes the form

$$E^{\ell,s}_{i,t} = Q^{\ell}_{i,t} F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}) \tag{2}$$

while production in a *renewable energy sector* $i \in \mathbb{I}\backslash\mathbb{I}_x$ in period t is given by

$$E^{\ell,s}_{i,t} = Q^{\ell}_{i,t} F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}). \tag{3}$$

Similar to (1), both specifications (2) and (3) allow for time- and region-specific productivity $Q^{\ell}_{i,t}$. In general, a higher productivity $Q^{\ell}_{i,t} > Q^{\ell'}_{i,t}$ may reflect a more developed technology in region ℓ relative to ℓ' or the fact that conditions to produce energy of type $i \in \mathbb{I}$ are more favorable in region ℓ than in ℓ' due to geographic conditions, etc.[4]

Technology and productivity

Denoting by $\mathbf{Q}_t := (Q^{\ell}_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0}$ the world productivity vector in period $t \geq 0$, we assume that the evolution of the sequence $(\mathbf{Q}_t)_{t\geq 0}$ is determined exogenously. The remainder imposes the following standard restrictions on production technologies (1), (2), and (3). The Inada condition ensures that each factor is employed in production.

Assumption 1. *Each production function $F_i : \mathbb{R}^{n_i}_+ \longrightarrow \mathbb{R}_+$, $i \in \mathbb{I}_0$ is linear homogeneous, strictly increasing, concave, and C^2 on $\mathbb{R}^{n_i}_{++}$. The first partial derivatives satisfy the Inada-condition $\lim_{z_m \searrow 0} \partial_{z_m} F_i(z_1, \ldots, z_{n_i}) = \infty \ \forall z = (z_1, \ldots, z_{n_i}) \in \mathbb{R}^{n_i}_{++}$ and $m = 1, \ldots, n_i$.*

[3] This abstracts from emissions generated from using renewable resources like biomass, etc. which are negligible relative to emissions from fossil fuels.

[4] For example, a solar energy plant located in the Sahara seems likely to produce more electricity output than an identical plant located in a northern European region like Norway while the opposite holds in the case with hydroelectric power generation.

Resource sectors

Resource sectors are uniquely identified by the energy sector $i \in \mathbb{I}_x$ which uses this resource in production. In each region $\ell \in \mathbb{L}$, there exists a single firm which extracts resources of type $i \in \mathbb{I}_x$ and supplies them to the global resource market. The amount of resource i extracted and supplied in period t is denoted $X_{i,t}^{\ell,s} \geq 0$ (to be distinguished from the amount $X_{i,t}^{\ell}$ demanded by energy sector $i \in \mathbb{I}_x$ in that region). Resource firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R_{i,0}^{\ell} \geq 0$ as a given parameter.[5] Feasible extraction plans are thus non-negative sequences $(X_{i,t}^{\ell,s})_{t\geq 0}$ which respect the feasibility constraint

$$\sum_{t=0}^{\infty} X_{i,t}^{\ell,s} \leq R_{i,0}^{\ell}. \tag{4}$$

To avoid trivialities, we impose the initial condition $\sum_{\ell\in\mathbb{L}} R_{i,0}^{\ell} > 0$, i.e., initial world resources are strictly positive for all $i \in \mathbb{I}_x$. It may, however, be the case that $R_{i,0}^{\ell} = 0$ in which case region ℓ does not own any resources of type i.

1.3. Climate model

Emissions of CO_2 are generated by using ('burning') exhaustible resources like coal, oil, and gas to produce energy. Thus, emissions occur at the energy stage in production. The amount of CO_2 generated by using one unit of exhaustible resource $i \in \mathbb{I}_x$ is physically determined by its carbon-content $\zeta_i \geq 0$. In particular, $\zeta_i = 0$ if the resource does not generate emissions, like uranium in the case of nuclear energy production.[6]

Summing the different types of exhaustible resource inputs weighted by their respective carbon content over all regions one obtains the total emissions of CO_2 in period t as

$$Z_t := \sum_{\ell\in\mathbb{L}} \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^{\ell}. \tag{5}$$

For the following analysis, we will adopt the climate model from Golosov et al. (2014), henceforth GHKT. However, none of our results depend on this particular specification.[7] Assume that the climate state in period t consists of permanent and non-permanent CO_2 in the atmosphere, denoted as $\mathbf{S}_t = (S_{1,t}, S_{2,t})$. Given the sequence of emissions $\{Z_t\}_{t\geq 0}$ determined by (5), the climate state evolves as

$$S_{1,t} = S_{1,t-1} + \phi_L Z_t \tag{6a}$$

$$S_{2,t} = (1-\phi)S_{2,t-1} + (1-\phi_L)\phi_0 Z_t \tag{6b}$$

Specification (6) assumes that a share $0 \leq \phi_L < 1$ of emissions become permanent CO_2. Out of the remaining emissions, a share ϕ_0 becomes non-permanent CO_2 which decays at constant rate

5 The assumption of constant extraction costs is a compromise between falling costs due to technological progress assumed in Golosov et al. (2014) and extraction costs which increase with the scarcity of the resource as in Acemoglu et al. (2012) and also discussed in Hotelling (1931). Our model should be amendable to extensions in either direction.

6 Our specification abstracts from emissions which occur at the resource stage when resources are extracted. While empirically such emissions certainly play a role, especially for uranium, they seem quantitatively negligible compared to emissions occurring at the energy stage on which we focus.

7 We show this formally in Section 6 where we adopt an alternative climate model from Gerlagh and Liski (2018).

$0 < \phi < 1$ while the remaining share $1 - \phi_0$ leaves the atmosphere (see GHKT for details). Total concentration of CO_2 at time t is thus given by

$$S_t = S_{1,t} + S_{2,t}. \tag{7}$$

Climate damages and temperature in period t depend exclusively on S_t. Denoting by $\bar{S} > 0$ the pre-industrial level of CO_2 in the atmosphere, climate damage in region ℓ is determined by a differentiable, strictly increasing function $D^\ell : [\bar{S}, \infty[\longrightarrow [0, 1[$,

$$D_t^\ell = D^\ell(S_t). \tag{8}$$

A specific functional form which will be used below is

$$D^\ell(S) = 1 - \exp\{-\gamma^\ell(S - \bar{S})\}, \quad \gamma^\ell > 0 \tag{9}$$

which corresponds to the choice in GHKT.[8] Regional differences in climate damage thus enter via region specific parameters γ^ℓ, $\ell \in \mathbb{L}$. In particular, the climate problem would be economically irrelevant if $\gamma^\ell \equiv 0$.

1.4. Consumption sector

The consumption sector in region $\ell \in \mathbb{L}$ consists of a single representative household which supplies labor and capital to the production process and decides about consumption and capital formation taking factor prices as given. In addition, the consumer is entitled to receive all profits from domestic firms and transfers from the government. Let K_0^ℓ denote initial capital in $t = 0$ and $N_t^{\ell,s} > 0$ the labor supplied in period t. The sequence $(\mathbf{N}_t^s)_{t \geq 0}$ of world labor supply $\mathbf{N}_t^s := (N_t^{\ell,s})_{\ell \in \mathbb{L}}$ is exogenously given in our model. The household's preferences over non-negative consumption sequences $(C_t^\ell)_{t \geq 0}$ are represented by a standard time-additive utility function

$$U((C_t^\ell)_{t \geq 0}) = \sum_{t=0}^{\infty} \beta^t u(C_t^\ell). \tag{10}$$

The subsequent analysis imposes the following restrictions on U.

Assumption 2. *The discount factor in (10) satisfies* $0 < \beta < 1$ *while u is of the form*

$$u(C) = \begin{cases} \frac{C^{1-\sigma}}{1-\sigma} & \textit{for} \quad \sigma > 0, \sigma \neq 1 \\ \log(C) & \textit{for} \quad \sigma = 1. \end{cases} \tag{11}$$

Assumption 2 will be key for the separability between efficiency and optimality derived in Section 3 and to determine the optimal climate policy in Section 4. Functional form (11) is precisely the class of utility functions consistent with balanced growth in our model with exogenous labor supply, cf. King et al. (1988). Thus, Assumption 2 is a standard restriction in the presence of exogenous productivity growth. It is also used in Nordhaus and Yang (1996) and almost any model of climate change and includes the logarithmic specification in GHKT or Hassler and Krusell (2012) as a special case.

[8] The general version of GHKT allows for γ to be time- and state-dependent. Here, we assume that it is constant, as they do in their numerical simulations, too.

1.5. Summary of the economy

The economy $\mathscr{E}$ introduced in the previous sections can be summarized by its regional and sectoral structure $\langle \mathbb{L}, \mathbb{I}_0, \mathbb{I}_x \rangle$, the production technologies $\langle (\mathbf{Q}_t)_{t\geq 0}, (F_i)_{i\in\mathbb{I}_0}, (c_i)_{i\in\mathbb{I}_x} \rangle$, consumer characteristics $\langle (\mathbf{N}_t^s)_{t\geq 0}, u, \beta \rangle$, and climate parameters $\langle (\zeta_i)_{i\in\mathbb{I}_x}, \phi, \phi_0, \phi_L \rangle$. In addition, initial values for capital supply $\mathbf{K}_0^s = (K_0^\ell)_{\ell\in\mathbb{L}}$, exhaustible resource stocks $\mathbf{R}_0 = (R_{i,0}^\ell)_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x}$, and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ are given.

2. Decentralized solution

This section studies the decentralized equilibrium solution of the economy where all producers and consumers behave optimally under perfect foresight and market clearing on all markets. All equilibrium variables are determined for a given *climate policy* which imposes a tax on emissions and distributes the tax revenue as transfers across regions.

2.1. Equilibrium prices

Unless stated otherwise, all prices in period t are denominated in units of time t consumption. As labor and energy outputs will be immobile across countries, their prices will, in general, be region-specific. Denote by $w_t^\ell > 0$ the wage and $p_{i,t}^\ell > 0$ the price per unit of energy type $i \in \mathbb{I}$ in region ℓ and period t. By contrast, capital and exhaustible resources are traded on international markets implying that their prices are not country-specific. The (rental) price of capital in period t is denoted as $r_t > 0$ and the world price of resource $i \in \mathbb{I}_x$ as $v_{i,t} > 0$.

Conceptually, all transactions take place in $t = 0$ and the consumption good in this period is chosen as the numeraire. Since the economy is deterministic, the price of time t consumption measured in units of consumption at time zero can be expressed as[9]

$$q_t = \prod_{s=1}^{t} r_s^{-1} \tag{12}$$

for each $t \geq 0$ where $q_0 = 1$. In the following analysis, the price defined in (12) serves as a discount factor which discounts payments in period t to period zero.

2.2. Climate policies

A *climate policy* consists of two parts. The first one is a *Carbon Tax Policy (CTP)* which levies a proportional tax on emissions. Taxes can vary over time and across regions.

Definition 1. A Carbon Tax Policy (CTP) is a non-negative sequence $\tau = (\tau_t)_{t\geq 0}$ where $\tau_t = (\tau_t^\ell)_{\ell\in\mathbb{L}}$ are the (region-specific) taxes to be paid per unit of CO_2 emitted in period $t \geq 0$.

Since emissions occur at the energy stage, taxes are paid by energy producers. Tax revenues are redistributed as lump-sum transfers to consumers. A natural restriction would be to assume that transfers equal tax revenue in each region. We will, however, adopt a more general setting

[9] In our deterministic model, this holds because $1/r_{t+1}$ is the price of a bond traded in period t that pays-off one unit of the consumption good at time $t + 1$. The prices in (12) are thus the Arrow-Debreu prices for this economy.

which does not require taxes and transfers to balance at the national level but allows for transfer payments across countries. This leads to the following concept of a *transfer policy* which determines the share of tax revenue received by each region. Such a transfer policy constitutes the second part of a climate policy.

Definition 2. A transfer policy is a mapping $\theta : \mathbb{L} \longrightarrow \mathbb{R}$, $\ell \mapsto \theta^\ell$ satisfying $\sum_{\ell\in\mathbb{L}} \theta^\ell = 1$ which determines the share of total tax revenue received by region $\ell \in \mathbb{L}$ in each period.

The pair (τ, θ) will be called a *climate policy*. Let T_t^ℓ denote the transfers received by consumers in region ℓ in period t. These transfers are determined by tax revenue and the given transfer policy as

$$T_t^\ell = \theta^\ell \sum_{k\in\mathbb{L}} \overbrace{\tau_t^k \cdot \underbrace{\sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^k}_{\text{Emissions in region } k}}^{\text{Tax revenue in region } k} \tag{13}$$

If $T_t^\ell > \tau_t^\ell \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^\ell$, region ℓ receives a net transfer from the other countries and contributes a net transfer otherwise. Note that the case $\theta^\ell < 0$ is not excluded in this definition, in which case consumers in region ℓ are taxed to finance transfers received by other countries. Thus, the previous specification also allows for international redistribution via lump-sum taxation. Moreover, the assumption that transfer shares are constant over time is without loss of generality, as the behavior of consumers derived below will exclusively depend on their lifetime transfer income defined as

$$T^\ell := \sum_{t=0}^{\infty} q_t T_t^\ell. \tag{14}$$

Thus, defining total discounted tax revenue

$$T := \sum_{t=0}^{\infty} q_t \sum_{k\in\mathbb{L}} \tau_t^k \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^k, \tag{15}$$

lifetime transfers satisfy

$$T^\ell = \theta^\ell T = \theta^\ell \sum_{t=0}^{\infty} q_t \sum_{k\in\mathbb{L}} \tau_t^k \sum_{i\in\mathbb{I}_x} \zeta_i X_{i,t}^k, \tag{16}$$

and we simply could have defined θ^ℓ as the ratio T^ℓ / T.

2.3. Producer behavior

Firms in each sector $i \in \mathbb{I}_0$ choose their productions plans to maximize the discounted stream of current and future profits. As there is no intertemporal linkage between these decisions, the decision problems of final and energy sectors can be formulated and solved on a period-by period basis. This, however, is not possible in the case of resource sectors which solve intertemporally dependent problems.

Final sector

Given damage-adjusted productivity and factor prices for labor, capital, and the list of energy prices $p_t^\ell = (p_{i,t}^\ell)_{i\in\mathbb{I}} \gg 0$, the final sector in region ℓ solves the following decision problem in period $t \geq 0$:

$$\max_{(K,N,E_1,\dots,E_I)\in\mathbb{R}_+^{2+I}} \left\{(1-D_t^\ell)Q_{0,t}^\ell F_0(K,N,(E_i)_{i\in\mathbb{I}}) - w_t^\ell N - r_t K - \sum_{i\in\mathbb{I}} p_{i,t}^\ell E_i\right\} \tag{17}$$

Under Assumption 1, any solution to (17) satisfies the following first order conditions which equate prices and marginal products of each production factor:

$$(1-D_t^\ell)Q_{0,t}^\ell \partial_K F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = r_t \tag{18a}$$

$$(1-D_t^\ell)Q_{0,t}^\ell \partial_N F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = w_t^\ell \tag{18b}$$

$$(1-D_t^\ell)Q_{0,t}^\ell \partial_{E_i} F_0(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}) = p_{i,t}^\ell \forall i \in \mathbb{I}. \tag{18c}$$

Energy sectors

A renewable energy sector $i \in \mathbb{I}\backslash\mathbb{I}_x$ in region $\ell \in \mathbb{L}$ takes sector specific productivity, factor prices for labor and capital, and the domestic energy price $p_{i,t}^\ell > 0$ as given and solves the following decision problem in each period $t \geq 0$:

$$\max_{(K,N)\in\mathbb{R}_+^2} \left\{p_{i,t}^\ell Q_{i,t}^\ell F_i(K,N) - w_t^\ell N - r_t K\right\}. \tag{19}$$

A solution to (19) is characterized by the following first order conditions

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i\left(K_{i,t}^\ell, N_{i,t}^\ell\right) = r_t \tag{20a}$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i\left(K_{i,t}^\ell, N_{i,t}^\ell\right) = w_t^\ell. \tag{20b}$$

An exhaustible energy sector $i \in \mathbb{I}_x$ takes, in addition, the resource price $v_{i,t} > 0$ and the tax $\tau_t^\ell \geq 0$ per unit of CO_2 as given parameters in the decision in period t. As each unit of resource i generates ζ_i units of CO_2, the decision problem in period t reads:

$$\max_{(K,N,X)\in\mathbb{R}_+^3} \left\{p_{i,t}^\ell Q_{i,t}^\ell F_i(K,N,X) - w_t^\ell N - r_t K - (v_{i,t} + \tau_t^\ell \zeta_i)X\right\}. \tag{21}$$

Clearly, the solution to (21) becomes independent of τ_t^ℓ if $\zeta_i = 0$, i.e., the firm employs a clean technology. The first order conditions associated with (21) are given by:

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i\left(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\right) = r_t \tag{22a}$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i\left(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\right) = w_t^\ell \tag{22b}$$

$$p_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i\left(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\right) = v_{i,t} + \zeta_i \tau_t^\ell. \tag{22c}$$

Resource sectors

The resource sector $i \in \mathbb{I}_x$ in region $\ell \in \mathbb{L}$ chooses a non-negative extraction sequence $(X_{i,t}^{\ell,s})_{t\geq 0}$ which satisfies the resource constraint (4). Resources extracted in period t are supplied to the global resource market. Given sequences of resource prices $(v_{i,t})_{t\geq 0}$, discount factors $(q_t)_{t\geq 0}$ defined by (12) and constant extraction costs $c_i \geq 0$, the resource sector maximizes the discounted sum of future profits. The decision problem reads

$$\max_{(X_{i,t}^{\ell,s})_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_{i,t}^{\ell,s} \mid X_{i,t}^{\ell,s} \geq 0 \, \forall t \geq 0, (4) \text{ holds} \right\}. \tag{23}$$

To avoid trivialities, assume $R_{i,0}^{\ell} > 0$. Then, the existence of optimal extraction plans determined as solutions to problem (23) can be characterized as follows.

Lemma 1. *If $R_{i,0}^{\ell} > 0$, the solution to (23) satisfies the following:*

(i) An interior solution $(X_t^*)_{t\geq 0} \gg 0$ exists if and only if resource prices satisfy

$$v_{i,t} - c_i = (v_{i,0} - c_i)/q_t \geq 0 \quad \forall t \geq 0. \tag{24}$$

(ii) If condition (24) is satisfied, there are two cases:
(a) If $v_{i,0} = c_i$, any sequence $(X_t^*)_{t\geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^{\ell}$ is a solution.
(b) If $v_{i,0} > c_i$, any sequence $(X_t^*)_{t\geq 0}$ satisfying $\sum_{t=0}^{\infty} X_t^* = R_{i,0}^{\ell}$ is a solution.

Condition (24) is a version of the classical Hotelling rule (cf. Hotelling, 1931) under which net resource prices must grow at the rate of interest for resource firms to be indifferent between extracting resources in different periods. Hassler and Krusell (2012) also derive a version of the Hotelling rule in their multi-region framework. One also observes from Lemma 1 (ii) that only in case (a) where $v_{i,t} = c_i$ for all $t \geq 0$ may it be optimal not to exhaust the entire stock of resources.

In either case, (24) permits maximum profits $\Pi_i^{\ell} := \sum_{t=0}^{\infty} q_t (v_{i,t} - c_i) X_t^*$ to be written as

$$\Pi_i^{\ell} = (v_{i,0} - c_i) R_{i,0}^{\ell}. \tag{25}$$

Intuitively, the discounted profit stream (25) of resource sector $i \in \mathbb{I}_x$ is the excess value of the initial stock of resources valued at time-zero prices net of extraction costs. Also note that given an optimal extraction plan $(X_{i,t}^{\ell,s})$ determined as a solution to (23), the period profit of resource sector $i \in \mathbb{I}_x$ in region $\ell \in \mathbb{L}$ is given by

$$\Pi_{i,t}^{\ell} = (v_{i,t} - c_i) X_{i,t}^{\ell,s} \geq 0. \tag{26}$$

In general, however, the quantity in (26) will be indeterminate at equilibrium due to the multiplicity of solutions to (23).

Equilibrium profits

All firms in region ℓ are owned by domestic consumers who are entitled to receive all profits. A direct consequence of Assumption 1 and the first order conditions derived in (18), (20), and (22) is that profits in final production and all energy sectors are zero. Thus, by (25) the total lifetime profit income of consumers in region $\ell \in \mathbb{L}$ is

$$\Pi^{\ell} = \sum_{i \in \mathbb{I}_x} \Pi_i^{\ell} = \sum_{i \in \mathbb{I}_x} (v_{i,0} - c_i) R_{i,0}^{\ell}. \tag{27}$$

2.4. Consumer behavior

Budget constraints

In each period $t \geq 0$, consumers in region $\ell \in \mathbb{L}$ receive labor income $w_t^{\ell} N_t^{\ell,s} > 0$, the return r_t on their current net asset holdings K_t^{ℓ}, profit income $\Pi_t^{\ell} \geq 0$, and transfers T_t^{ℓ}. Their choices of current consumption $C_t^{\ell} \geq 0$ and investment K_{t+1}^{ℓ} satisfy the period budget constraint

$$C_t^\ell + K_{t+1}^\ell = r_t K_t^\ell + w_t^\ell N_t^{\ell,s} + T_t^\ell + \Pi_t^\ell \quad \forall t \geq 0. \tag{28}$$

At the individual level, capital investment may be negative[10] but must satisfy the No-Ponzi game condition

$$\lim_{t\to\infty} q_t K_{t+1}^\ell \geq 0 \tag{29}$$

requiring consumers to ultimately repay any outstanding debt. Using (28) and (29) one can recursively eliminate investment to obtain the consumer's lifetime budget constraint

$$\sum_{t=0}^{\infty} q_t C_t^\ell \leq W^\ell + T^\ell. \tag{30}$$

Here, T^ℓ denotes lifetime transfer income defined in (14) and

$$W^\ell := r_0 K_0^\ell + \Pi^\ell + \sum_{t=0}^{\infty} q_t w_t^\ell N_t^{\ell,s} \tag{31}$$

is the consumer's lifetime non-transfer income determined by initial asset holdings K_0^ℓ, lifetime profit income Π^ℓ determined by (27), and lifetime labor income.

Optimal consumption plans

The consumer chooses a consumption sequence $(C_t^\ell)_{t\geq 0}$ to maximize lifetime utility (10) subject to her lifetime budget constraint. The decision problem reads:

$$\max_{(C_t^\ell)_{t\geq 0}} \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t^\ell) \,\middle|\, C_t^\ell \geq 0 \; \forall t \geq 0, \text{(30) holds} \right\}. \tag{32}$$

Equation (30) shows that existence of a solution to (32) requires the solvency condition

$$T^\ell > -W^\ell \tag{33}$$

imposing a lower bound on transfers which becomes relevant with taxation ($\theta^\ell < 0$). Standard (variational) arguments imply that any solution $(C_t^{\ell *})_{t\geq 0}$ to (32) must satisfy the Euler equations

$$r_{t+1}\beta u'(C_{t+1}^\ell) = u'(C_t^\ell) \quad \forall t \geq 0, \tag{34}$$

and the lifetime budget constraint (30) holds with equality. The latter is equivalent to the transversality condition

$$\lim_{t\to\infty} q_t K_{t+1}^\ell = 0. \tag{35}$$

In fact, the restriction imposed by Assumption 2 allows us to characterize the unique solution to (32) in the following lemma.

[10] This assumption can be justified by assuming that an international bond market coexists with the capital market on which borrowing and lending takes place. The consumer in period t chooses capital investment $\tilde{K}_{t+1}^\ell \geq 0$ and bond purchases B_{t+1}^ℓ subject to the budget constraint (28). Market clearing requires $K_{t+1} = \sum_{\ell\in\mathbb{L}} \tilde{K}_{t+1}^\ell$ on the capital market and $\sum_{\ell\in\mathbb{L}} B_{t+1}^\ell = 0$ for the bond market. Setting $K_{t+}^\ell := \tilde{K}_{t+1}^\ell + B_{t+1}^\ell$ then implies the equilibrium conditions derived below.

Lemma 2. *Let Assumption 2 hold and the solvency condition (33) be satisfied. Then, problem (32) has a unique solution $(C_t^{\ell *})_{t\geq 0}$ given by*

$$C_t^{\ell *} = \frac{\left(\beta^t/q_t\right)^{\frac{1}{\sigma}}\left[W^\ell + T^\ell\right]}{\sum_{s=0}^{\infty} q_s\left(\beta^s/q_s\right)^{\frac{1}{\sigma}}} \qquad t \geq 0. \tag{36}$$

Optimal consumption behavior defined by (36) thus satisfies the permanent income hypothesis by consuming a fraction of lifetime income including transfers in each period. Note that this behavior requires the possibility of unconstrained borrowing and lending in a frictionless capital market.

2.5. Market clearing

Restrictions on trade

Trade between countries occurs on global markets for capital, final output, and exhaustible resources of each type $i \in \mathbb{I}_x$. All these goods can freely be exported without additional costs. As there is no sign restriction on capital investment at the individual level, consumers can also take loans permitting intertemporal borrowing and lending of final output between regions. By contrast, labor supply is immobile and can only be employed in domestic production sectors. Likewise, energy goods and services can only be used in domestic final good production.[11] Thus, there are domestic markets for labor and energy of all types $i \in \mathbb{I}$ in each region $\ell \in \mathbb{L}$.

Domestic markets

Market clearing on the domestic labor market in region ℓ in period $t \geq 0$ requires

$$\sum_{i\in\mathbb{I}_0} N_{i,t}^\ell \overset{!}{=} N_t^{\ell,s}. \tag{37}$$

Since energy is non-tradable across countries, energy demanded in final production must coincide with domestic energy production in each region. The market clearing condition for energy type $i \in \mathbb{I}$ in region $\ell \in \mathbb{L}$ which must hold in each period t is therefore simply

$$E_{i,t}^\ell \overset{!}{=} E_{i,t}^{\ell,s}. \tag{38}$$

International markets

Let $K_t := \sum_{\ell\in\mathbb{L}} K_t^\ell$ be the aggregate stock of productive capital supplied to production in period t. Market clearing on the world capital market in period t requires

$$\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_0} K_{i,t}^\ell \overset{!}{=} K_t. \tag{39}$$

Since period profit income (26) is, in general, indeterminate at equilibrium, so is consumers' individual net capital position K_t^ℓ, $\ell \in \mathbb{L}$ in the period budget constraint (28).

The market clearing condition for exhaustible resource $i \in \mathbb{I}_x$ in period t reads

$$\sum_{\ell\in\mathbb{L}} X_{i,t}^{\ell,s} \overset{!}{=} \sum_{\ell\in\mathbb{L}} X_{i,t}^\ell. \tag{40}$$

[11] In fact, this is one reason why we disentangle the energy stage and the resource stage in our model, compared to GHKT.

If $X_{i,t}^{\ell} < X_{i,t}^{\ell,s}$ region ℓ is a net exporter of resource $i \in \mathbb{I}_x$ in period t and a net importer otherwise. Summing (4) over all countries and using (40), the allocation of resources across production sectors must satisfy the world resource constraint

$$\sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell} \leq R_{i,0} \tag{41}$$

where $R_{i,0} := \sum_{\ell \in \mathbb{L}} R_{i,0}^{\ell}$ is the total initial stock of resource i. An immediate consequence of Lemma 1 is that the equilibrium amount of resources $X_{i,t}^{\ell,s}$ supplied by region ℓ and period profits (26) will, in general, be indeterminate. As resources extracted in different countries are perfect substitutes, however, the equilibrium extraction plans can always be chosen compatible with the resource constraint (4) in each region.

Finally, summing the consumers' period budget constraints (28) over all regions and exploiting the first order and zero profit conditions for all sectors $i \in \mathbb{I}_0$ in conjunction with (26) and the market clearing conditions (37), (38), (39), and (40) together with (13) one obtains the market clearing condition for final output in period t as

$$K_{t+1} + \sum_{\ell \in \mathbb{L}} C_t^{\ell} + \sum_{i \in \mathbb{I}_x} c_i \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell} = \sum_{\ell \in \mathbb{L}} Y_t^{\ell}. \tag{42}$$

Here, K_{t+1} is productive capital formed in period t and supplied to production in $t+1$.

2.6. Equilibrium

For purposes of a compact notation, we employ the following vector notation for the variables introduced in the previous sections for each t:

$$\begin{array}{lllllllll}
\mathbf{Q}_t & := & (Q_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{N}_t^s & := & (N_t^{\ell,s})_{\ell\in\mathbb{L}} & \mathbf{C}_t & := & (C_t^{\ell})_{\ell\in\mathbb{L}} \\
\mathbf{Y}_t & := & (Y_t^{\ell})_{\ell\in\mathbb{L}} & \mathbf{E}_t & := & (E_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{X}_t & := & (X_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x} \\
\mathbf{N}_t & := & (N_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{K}_t & := & (K_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_0} & \mathbf{S}_t & := & (S_{t,1}, S_{t,2}) \\
\mathbf{w}_t & := & (w_t^{\ell})_{\ell\in\mathbb{L}}, & \mathbf{p}_t & := & (p_{i,t}^{\ell})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}} & \mathbf{v}_t & := & (v_{i,t})_{i\in\mathbb{I}_x}.
\end{array} \tag{43}$$

All variables defined in (43) take values in the appropriate positive orthant of $\mathbb{R}^n$.

Definition of equilibrium

Let the climate tax policy τ and transfer policy θ defined as above be given. The following definition of equilibrium is standard. Here and in the remainder we denote equilibrium variables by a * superscript.

Definition 3. Given tax policy τ and transfer policy θ, an equilibrium of $\mathscr{E}$ is an allocation $\mathbf{A}^* = \left(\mathbf{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*\right)_{t\geq 0}$ and prices $\mathbf{P}^* = \left(r_t^*, \mathbf{w}_t^*, \mathbf{p}_t^*, \mathbf{v}_t^*\right)_{t\geq 0}$ such that:

(i) The allocation is consistent with the production technologies (1), (2), and (3) and the market clearing conditions/resource constraints (37), (38), (39), (41), and (42).
(ii) Producers behave optimally, i.e., equations (18), (22), and (20) hold for all $t \geq 0$. Profits of resource firms are given by (25) while resource prices evolve as in (24).
(iii) Consumers behave optimally as described in Lemma 2 with profit incomes determined by (27) and transfers satisfying (16) and the solvency condition (33).
(iv) Climate variables evolve according to (6) with emissions given by (5) and climate damages in (1) determined by (7) and (8).

Properties of equilibrium

If we want to emphasize the dependence of the equilibrium allocation on policy variables, we will write $\mathbf{A}^*(\tau, \theta)$, etc. A special case of Definition 3 is the Laissez faire equilibrium with no taxation, i.e., $\tau \equiv 0$. The induced equilibrium allocation $\mathbf{A}^{\mathrm{LF}} := \mathbf{A}^*(0, \theta)$ is independent of θ and constitutes an important benchmark in the subsequent discussion.[12] It is clear that this solution will, in general not constitute a Pareto optimal outcome due to the climate externality in production.

The following results establish additional properties of equilibrium that follow from our restrictions on technologies and preferences. As before, we denote by $R_{i,0} = \sum_{\ell \in \mathbb{L}} R_{i,0}^{\ell}$ to be the total initial stock of resource $i \in \mathbb{I}_x$.

Lemma 3. *Under Assumptions 1 and 2, any equilibrium allocation has the following properties:*

(i) The allocation is interior, i.e., $\mathbf{A}^* \gg 0$.

(ii) Consumption $C_t^{\ell *}$ *is a constant share of world consumption* $\bar{C}_t^* := \sum_{\ell \in \mathbb{L}} C_t^{\ell *}$, *i.e.,*

$$C_t^{\ell *} = \mu^{\ell *} \bar{C}_t^* \tag{44}$$

for all $\ell \in \mathbb{L}$ *and* $t \geq 0$ *where* $\mu^{\ell *} > 0$ *and* $\sum_{\ell \in \mathbb{L}} \mu^{\ell *} = 1$.

(iii) Prices and extraction of each resource $i \in \mathbb{I}_x$ *satisfy the following:*

(a) If $R_{i,0} = \infty$, *then* $v_{i,0} = c_i$ *implying* $v_{i,t} \equiv c_i$.

(b) If $R_{i,0} < \infty$, *then* $\lim_{t \to \infty}(v_{i,t} + \zeta_i \tau_t^{\ell}) = \infty$.

(c) If $R_{i,0} > \sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell *}$, *then* $v_{i,0} = c_i$ *and* $\lim_{t \to \infty}(v_{i,t} + \zeta_i \tau_t^{\ell}) = \infty$.

Lemma 3 (ii) shows that consumption in each region moves in lock-step with world consumption over time. This property is due to consumer utility restricted by Assumption 2 and will play a key role to characterize how policy affects equilibrium allocations.

The result in (iii) shows that an infinite stock of resources drives resource prices down to extraction costs. Hence, there is no scarcity rent on that resource and profits are zero.[13] Conversely, if the resource stock is finite, gross resource prices (including taxes) must converge to infinity which requires $v_{i,0} > c_i$ in the absence of taxation. Moreover, (iii) shows that each resource is completely exhausted unless its scarcity rent is zero and taxes on emissions grow infinitely large. Thus, aggressive taxation is needed to prevent resources from being completely exhausted. In particular, all resources are fully exhausted at the Laissez faire equilibrium. See Hillebrand and Hillebrand (2018) for further discussion and application of these results in a numerical study of the model.

The final result of this section disentangles the separate impact of the tax policy τ and the transfer policy θ on the equilibrium allocation $\mathbf{A}^*(\tau, \theta)$. Essentially, it shows that transfers only determine consumption in each region as a constant share of world consumption but are irrelevant

[12] Our subsequent discussion and notation presume that each policy considered induces a unique equilibrium. A general proof of existence/uniqueness of equilibrium is beyond the scope of this paper. The recursive structure of equilibria and their computation is discussed in detail in Hillebrand and Hillebrand (2018) suggesting that the uniqueness assumption is justified.

[13] An infinite resource stock is typically justified by assuming the existence of a backstop technology which provides an equivalent substitute for the resource in the future. Such a backstop technology is implicitly assumed in GHKT for coal.Hillebrand and Hillebrand (2018) provide a critical discussion of this assumption and show that it is key to obtain the quantitative predictions in GHKT.

for all other equilibrium variables. This result will play a major role in Section 4 when we study climate policies which implement the social optimum as an equilibrium allocation.

Proposition 1. *Given a climate policy* (τ, θ)*, let* $\mathbf{A}^* = \left(\mathbf{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*\right)_{t\geq 0}$ *be the induced equilibrium allocation and* $\mathbf{P}^*$ *the equilibrium price system. For each* $t \geq 0$*, let* $\bar{C}_t^* = \sum_{\ell\in\mathbb{L}} C_t^{\ell *}$ *denote aggregate equilibrium consumption. Then, the following holds:*

(i) The tax policy τ *determines the aggregate equilibrium allocation*

$$\bar{\mathbf{A}}^* = \left(\bar{C}_t^*, K_{t+1}^*, \mathbf{Y}_t^*, \mathbf{E}_t^*, \mathbf{K}_t^*, \mathbf{N}_t^*, \mathbf{X}_t^*, \mathbf{S}_t^*\right)_{t\geq 0} \tag{45}$$

and the price system $\mathbf{P}^*$ *which are both independent of the transfer policy* θ.

(ii) The transfer policy θ *only affects the distribution of aggregate consumption across regions, i.e., the consumption shares* $\mu^{\ell *}$ *in (44) which take the form*

$$\mu^{\ell *} = \frac{W^{\ell *} + \theta^\ell T^*}{\sum_{k\in\mathbb{L}} W^{k*} + T^*}, \tag{46}$$

with non-transfer incomes $W^{\ell *}$ *determined by (31) and tax revenue* T^* *by (15).*

The aggregate allocation $\bar{\mathbf{A}}^*$ in (45) determines world consumption but does not specify its distribution across regions. Apart from that, it contains the same variables as the equilibrium allocation $\mathbf{A}^*$. The equilibrium distribution of consumption determined by (46) corresponds to the relative sizes of consumers' lifetime incomes including transfers. Only this equilibrium quantity depends on the transfer policy. Note that consumption shares become independent of θ along the Laissez faire equilibrium where $T^* = 0$.

3. Optimal solution

In this section we determine an *optimal allocation* as the solution to a planning problem (PP) which maximizes consumer utility subject to the feasibility constraints imposed by technology, resources, and climate change. The major difference to the decentralized solution is that the planning problem incorporates the climate externality and the link between emissions, climate damage, and productivity in final good production.

In a multi-region world, there is no unique choice of the planner's objective function which must necessarily incorporate the trade-offs between the interests of different countries. The standard approach in the literature (see, e.g., Nordhaus and Yang, 1996) also adopted here aggregates utilities in different countries based on a weighting scheme which assigns a certain weight to the utility of consumers in each region. Such a weighting scheme is essentially equivalent to choosing certain minimum utility levels for all countries $\ell \neq 1$ and then maximizing utility of region 1 as is done in Eyckmans and Tulkens (2003) to obtain a Pareto-optimal allocation.

A major advantage of our restrictions on preferences in Assumption 2 is that it gives rise to a separation result which permits to compute an optimal allocation in two steps. First, we determine an *efficient allocation* which maximizes utility of a fictitious world representative consumer. This efficient solution completely specifies the optimal climate path and the entire allocation of production factors and resources across countries together with aggregate world consumption. Most importantly, the efficient solution is *independent* of the employed weighting scheme. In a second step, we determine an *optimal distribution* of world consumption across different countries

to obtain an *optimal allocation* which maximizes a weighted utility index reflecting the trade-off between the interests of different countries. This separability between efficiency and optimal distribution will be the key to determine an optimal climate policy in the next chapter. In fact, this separation result has been well-known in the literature on public goods. A finite-dimensional version was first proved by Bergstrom and Cornes (1983).

3.1. Optimal allocations

Feasible allocations

Consider a planner who chooses a feasible world allocation subject to the restrictions imposed by technology, factor mobility, and resource constraints. Formally, using the notation introduced in the previous section, the planner takes the sequences of productivity $(\mathbf{Q}_t)_{t\geq 0}$ and labor supply $(\mathbf{N}_t^s)_{t\geq 0}$ as given. In addition, initial world capital $K_0 > 0$, the initial world stock $R_{i,0} > 0$ of each exhaustible resource $i \in \mathbb{I}_x$, and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ are given as well. While final output, capital, and exhaustible resources can freely be allocated across countries, labor and energy outputs can only be used within each region. Thus, the planner faces essentially the same restrictions as producers and consumers in the decentralized solution including constraints (37), (38), (39), and (42) when allocating labor, capital, and output. Further, the given initial stock of world resources imposes the restrictions (41) on the use of exhaustible resource $i \in \mathbb{I}_x$ in production.[14] This leads to the following definition of a feasible allocation.

Definition 4.

(i) A feasible allocation is a sequence $\mathbf{A} = \big(\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t\big)_{t\geq 0}$ which satisfies (1), (2), (3), (5), (6), (7), (37), (38), (39), (41), and (42) for all $t \geq 0$.
(ii) The set of feasible allocations of the economy $\mathscr{E}$ is denoted $\mathbb{A}$.

In particular, equilibrium allocations studied in the previous section are feasible, i.e., $\mathbf{A}^* \in \mathbb{A}$.

Objective function

The distribution of consumption across countries necessarily induces a trade-off between the utility levels attained by consumers in different countries. To incorporate this trade-off, assume that the planner uses a weighting scheme corresponding to a list of utility weights $\omega = (\omega^\ell)_{\ell\in\mathbb{L}}$ where $\omega^\ell \geq 0$ represents the weight attached to consumer utility in region ℓ in the planner's decision. Formally, we have

Definition 5. A utility weighting scheme is a map $\omega : \mathbb{L} \longrightarrow \mathbb{R}_+$, $\ell \mapsto \omega^\ell$ which satisfies $\sum_{\ell\in\mathbb{L}} \omega^\ell = 1$.

In what follows, let $\Delta := \{(x_1, \ldots, x_L) \in \mathbb{R}^L \,|\, \sum_{\ell=1}^L x_\ell = 1\}$ denote the unit-simplex in $\mathbb{R}^L$. Then, the set of all possible weighting schemes can be identified with $\Delta_+ := \Delta \cap \mathbb{R}^L_+$. Each weighting scheme $\omega \in \Delta_+$ defines the following weighted utility index

[14] As resources extracted in different countries are perfect substitutes, the solution to the planning problem does not determine where these resources are extracted. However, any allocation of exhaustible resources $(X^\ell_{i,t})_{(\ell,i)\in\mathbb{L}\times\mathbb{I}_x})_{t\geq 0}$ satisfying the feasibility constraint (41) can always be chosen compatible with the individual resource constraints (4) in each region.

$$V((\mathbf{C}_t)_{t\geq 0};\omega) := \sum_{t=0}^{\infty}\beta^t \sum_{\ell\in\mathbb{L}}\omega^\ell u(C_t^\ell) \tag{47}$$

which depends on world consumption $(\mathbf{C}_t)_{t\geq 0}$ where $\mathbf{C}_t = (C_t^\ell)_{\ell\in\mathbb{L}}$ for all $t \geq 0$.

Weighted Planning Problem

Given $\omega \in \Delta_+$, use (47) to define the following **Weighted Planning Problem (WPP)**

$$\max_{\mathbf{A}}\left\{ V((\mathbf{C}_t)_{t\geq 0};\omega) \,\middle|\, \mathbf{A} = (\mathbf{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t\geq 0} \in \mathbb{A}\right\}. \tag{48}$$

Assuming it exists and is unique, denote the solution to (48) as

$$\mathbf{A}^{\text{opt}} = \left(\mathbf{C}_t^{\text{opt}}, K_{t+1}^{\text{opt}}, \mathbf{Y}_t^{\text{opt}}, \mathbf{E}_t^{\text{opt}}, \mathbf{K}_t^{\text{opt}}, \mathbf{N}_t^{\text{opt}}, \mathbf{X}_t^{\text{opt}}, \mathbf{S}_t^{\text{opt}}\right)_{t\geq 0}. \tag{49}$$

It is clear that, in general, the solution to (48) will depend on the weighting scheme ω. Thus, we will write $\mathbf{A}^{\text{opt}}(\omega)$ as a way of emphasizing this dependence. It is also clear that for any weighting scheme $\omega \in \Delta_+$ the solution $\mathbf{A}^{\text{opt}}(\omega)$ to (48) is a Pareto-optimum on the set of feasible allocations $\mathbb{A}$.

3.2. *Efficient aggregate allocations*

Feasible aggregate allocations

Consider now a modified planning problem which faces the same restrictions as before but does not specify the distribution of consumption across different countries. As before, denote aggregate world consumption in period t by $\bar{C}_t \geq 0$ and write the resource constraint (42) as

$$K_{t+1} + \bar{C}_t + \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x} c_i X_{i,t}^\ell = \sum_{\ell\in\mathbb{L}} Y_t^\ell. \tag{50}$$

Replacing (42) by (50) leads to the following definition of a feasible aggregate allocation which specifies aggregate world consumption but not its distribution across countries. Apart from that, it involves the same variables as a feasible allocation defined above.

Definition 6.

(i) A feasible aggregate allocation is a sequence $\bar{\mathbf{A}} = \left(\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t\right)_{t\geq 0}$ which satisfies (1), (2), (3), (5), (6), (7), (37), (38), (39), (41), (50) for all $t \geq 0$.

(ii) The set of feasible aggregate allocations of the economy $\mathscr{E}$ is denoted $\bar{\mathbb{A}}$.

In particular, the aggregate equilibrium allocation in (45) is feasible, i.e., $\bar{\mathbf{A}}^* \in \bar{\mathbb{A}}$.

Aggregate planning problem

The following modified planning problem maximizes utility of a fictitious world representative consumer who consumes $\bar{C}_t$ in period t and has the same utility function as consumers in each region. This leads to the following **Aggregate Planning Problem (APP)**:

$$\max_{\bar{\mathbf{A}}}\left\{ \sum_{t=0}^{\infty}\beta^t u(\bar{C}_t) \,\middle|\, \bar{\mathbf{A}} = (\bar{C}_t, K_{t+1}, \mathbf{Y}_t, \mathbf{E}_t, \mathbf{K}_t, \mathbf{N}_t, \mathbf{X}_t, \mathbf{S}_t)_{t\geq 0} \in \bar{\mathbb{A}}\right\}. \tag{51}$$

A solution to (51) will be denoted

$$\bar{\mathbf{A}}^{\text{eff}} = (\bar{C}_t^{\text{eff}}, K_{t+1}^{\text{eff}}, \mathbf{Y}_t^{\text{eff}}, \mathbf{E}_t^{\text{eff}}, \mathbf{K}_t^{\text{eff}}, \mathbf{N}_t^{\text{eff}}, \mathbf{X}_t^{\text{eff}}, \mathbf{S}_t^{\text{eff}})_{t\geq 0} \tag{52}$$

and referred to as an *efficient aggregate allocation*. An efficient solution $\bar{\mathbf{A}}^{\text{eff}}$ specifies the entire allocation of production factors and resources across countries but leaves undetermined the distribution of aggregate consumption across countries. It is therefore independent of any weights attached to the interests of different countries and constitutes the main ingredient to our separation result.

3.3. Optimal distribution of consumption

To prepare the main result of this section, let a weighting scheme $\omega \in \Delta_+$ be arbitrary but fixed and consider the static problem of distributing a given level $\bar{C} > 0$ of aggregate consumption across countries in an arbitrary period. The decision variable is a *consumption distribution* $\mu = (\mu^\ell)_{\ell\in\mathbb{L}}$ where $\mu^\ell \geq 0$ is the share of $\bar{C}$ given to region ℓ. Since $\sum_{\ell\in\mathbb{L}} \mu^\ell = 1$, any feasible consumption distribution satisfies $\mu \in \Delta_+$ defined as above. An optimal consumption distribution $\mu^{\text{opt}} = (\mu^{\ell,\text{opt}})_{\ell\in\mathbb{L}}$ can be determined as the solution to the problem

$$\max_{\mu=(\mu^\ell)_{\ell\in\mathbb{L}}} \left\{ \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell \bar{C}) \,\middle|\, \mu^\ell \geq 0 \; \forall \ell \in \mathbb{L}, \sum_{\ell\in\mathbb{L}} \mu^\ell \leq 1 \right\}. \tag{53}$$

The following lemma establishes that (53) has a unique solution which can be computed explicitly and which, crucially, is independent of $\bar{C}$. The result in (ii) establishes an essential equivalence between the choice of a weighting scheme ω and a consumption distribution μ which will be exploited in Section 4.3.

Lemma 4. *Let u be of the form (11) from Assumption 2. Then, the following holds:*

(i) For each weighting scheme $\omega = (\omega^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$, there exists a unique consumption distribution $\mu^{\text{opt}}(\omega) = (\mu^{\ell,\text{opt}}(\omega))_{\ell\in\mathbb{L}}$ which solves (53) taking the form

$$\mu^{\ell,\text{opt}}(\omega) = \frac{(\omega^\ell)^{\frac{1}{\sigma}}}{\sum_{k\in\mathbb{L}} (\omega^k)^{\frac{1}{\sigma}}}, \quad \ell \in \mathbb{L}. \tag{54}$$

(ii) For any consumption distribution $\mu = (\mu^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$ there exists a weighting scheme $\omega \in \Delta_+$ which rationalizes μ in the sense that $\mu = \mu^{\text{opt}}(\omega)$ solves (53).

3.4. From efficiency to optimality

The main result of this section shows that the solution (49) to the WPP (48) can be obtained from the efficient allocation (52) by distributing aggregate consumption optimally across countries. Moreover, for any weighting scheme ω, the optimal distribution is time-invariant and can be determined as the solution to (53). Using Lemma 4 allows us to state this result in the following theorem.

Theorem 1. *Let Assumptions 1 and 2 be satisfied and define the efficient allocation $\mathbf{A}^{\text{eff}}$ as in (52). Given a weighting scheme ω, let $\mu^{\text{opt}}(\omega) = (\mu^{\ell,\text{opt}}(\omega))_{\ell\in\mathbb{L}}$ solve (53). Then, the allocation*

$$\mathbf{A} = (\mu^{\text{opt}}(\omega)\bar{C}_t^{\text{eff}}, K_{t+1}^{\text{eff}}, \mathbf{Y}_t^{\text{eff}}, \mathbf{E}_t^{\text{eff}}, \mathbf{K}_t^{\text{eff}}, \mathbf{N}_t^{\text{eff}}, \mathbf{X}_t^{\text{eff}}, \mathbf{S}_t^{\text{eff}})_{t\geq 0} \tag{55}$$

solves the WPP (48), i.e., $\mathbf{A} = \mathbf{A}^{\text{opt}}(\omega)$.

In words, Theorem 1 says that a solution to the WPP can simply be obtained from the efficient solution (52) by distribution aggregate consumption in an optimal fashion determined by the weighting scheme ω. In particular, the entire allocation of production factors, resources, and emissions is completely determined by the efficient solution.

As its major implication, the previous result allows to compute a unique efficient allocation of production factors, resources, emissions, climate damages, etc. which is completely independent of the weights that the interests of different countries receive in the decision. The weighting scheme is therefore irrelevant for answering the question what the optimal climate path is and *where* and *how* emissions should be reduced. Intuitively, there exists a unique allocation of factors and resources across regions to produce an efficient world consumption sequence which incorporates the climate externality. The weighting scheme becomes relevant only to determine how this efficiently produced world consumption sequence should be distributed across regions. An optimal consumption distribution can be computed using (54) once a suitable weighting scheme has been chosen and, by Lemma 4 (ii) is in fact equivalent to such a choice.

3.5. Computing the efficient allocation

The previous results show that the social optimum is essentially characterized by the efficient solution (52). Adopting a standard infinite-dimensional Lagrangian approach, it is now straightforward to obtain explicit conditions which completely characterize this solution. Detailed computations can be found in Section A.7 in the appendix. The main findings are as follows.

Social cost of carbon

The total costs Λ_t of emitting one additional unit of CO_2 in period t (measured in units of time t consumption) correspond to the discounted sum of all future marginal climate damages in all regions caused by this emission. Formally,

$$\Lambda_t = \sum_{n=0}^{\infty} \underbrace{\beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)}}_{\text{discount factor}} \underbrace{\Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\Big)}_{\text{climate parameters}}$$
$$\times \sum_{\ell \in \mathbb{L}} \underbrace{D^{\ell'}(S_{t+n}) Q^{\ell}_{0,t} F_0\big(K^{\ell}_{0,t}, N^{\ell}_{0,t}, (E^{\ell}_{i,t})_{i\in\mathbb{I}}\big)}_{\text{marginal damage in region } \ell}. \tag{56}$$

Equation (56) is a multi-region version of the result in GHKT. Note that Λ_t is independent of ℓ and i and depends on the structural parameters of the model and endogenous model variables in a complicated way, unless stronger restrictions similar to those in GHKT are imposed.[15] The term (56) is the key quantity to incorporate the climate externality into the (shadow) price of exhaustible resources.

Efficiency conditions

The remaining optimality conditions essentially ensure *intratemporal* and *intertemporal efficiency* in production and resource extraction in each period $t \geq 0$. Denote by

$$\hat{p}^{\ell}_{i,t} = (1-D^{\ell}_t) Q^{\ell}_{0,t} \partial_{E_i} F_0\big(K^{\ell}_{0,t}, N^{\ell}_{0,t}, \mathbf{E}^{\ell}_t\big), \tag{57}$$

[15] Climate costs Λ_t are independent of ℓ because future climate damages are discounted by the same discount factors in each region. For a different setup with region-specific discount rates see Eyckmans and Tulkens (2003).

the time t *shadow price* of energy type $i \in \mathbb{I}$ in region $\ell \in \mathbb{L}$ (measured in units of time t consumption units). Using (57), marginal products of capital are equalized across all countries and sectors, i.e., for all $\ell, \ell' \in \mathbb{L}$ and all $i \in \mathbb{I}$[16]:

$$(1 - D^\ell(S_t))Q^\ell_{0,t}\partial_K F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, \mathbf{E}^\ell_t\big) = \hat{p}^{\ell'}_{i,t} Q^{\ell'}_{i,t}\partial_K F_i(K^{\ell'}_{i,t}, N^{\ell'}_{i,t}, X^{\ell'}_{i,t}) \tag{58}$$

Second, in each region $\ell \in \mathbb{L}$ marginal products of labor are equalized across all sectors (although not across countries due to labor immobility), i.e., for all $i \in \mathbb{I}$

$$(1 - D^\ell(S_t))Q^\ell_{0,t}\partial_N F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}\big) = \hat{p}^{\ell}_{i,t} Q^{\ell}_{i,t}\partial_N F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}). \tag{59}$$

Third, for all $i \in \mathbb{I}_x$, the constraint (41) is binding and resource extraction is intratemporally efficient in each period $t \geq 0$, i.e., for all $\ell \in \mathbb{L}$:

$$\hat{p}^{\ell}_{i,t} Q^{\ell}_{i,t}\partial_X F_i(K^{\ell}_{i,t}, N^{\ell}_{i,t}, X^{\ell}_{i,t}) - \zeta_i \Lambda_t = \hat{v}_{i,t}. \tag{60}$$

Equation (60) defines the true shadow price of resources as the marginal product in production minus the cost of emissions defined in (56). Compared to the laissez-faire equilibrium allocation, the social planner includes a wedge between the marginal product of dirty resource i and its (shadow) price which accounts for the externality cost of an additional unit of emissions. This is the key difference to the equilibrium condition (22c) along the Laissez faire equilibrium which fails to take this cost into account. If $D^\ell \equiv 0$, however, the efficient solution coincides with the aggregate equilibrium allocation (45).

Intertemporal efficiency of final good allocation (consumption vs. capital formation) is ensured by the standard Euler equation which holds for all $\ell \in \mathbb{L}$ and $t \geq 1$:

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t)(1 - D^\ell(S_t))Q^\ell_{0,t}\partial_K F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, (E^\ell_{i,t})_{i\in\mathbb{I}}\big). \tag{61}$$

Condition (61) in conjunction with (58) also equates (implicit) capital returns across countries in each period.

Defining the shadow price of resource extraction as in (60), intertemporally efficient extraction of resource $i \in \mathbb{I}_x$ is ensured by the condition:

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)}\Big(\hat{v}_{i,t+1} - c_i\Big). \tag{62}$$

Finally, standard arguments also require the transversality condition

$$\lim_{t\to\infty} \beta^t u'(\bar{C}_t) K_{t+1} = 0. \tag{63}$$

The following proposition summarizes the result of this section. The proof is given in Section A.7 in the appendix.

Proposition 2. *Let Assumptions 1 and 2 hold. Then, any feasible aggregate allocation $\bar{\mathbf{A}} \in \bar{\mathbb{A}}$ which satisfies conditions (56)–(62) for all $t \geq 0$ as well as (63) is efficient, i.e., solves (51).*

[16] Here and in the sequel, we incur a slight abuse of notation by including $X^\ell_{i,t}$ as a 'dummy' argument of F_i even if $i \notin \mathbb{I}_x$ to save some notation.

4. Optimal climate policies

This section determines a climate policy which implements the ω-optimal allocation (49) as an equilibrium allocation in the sense of Definition 3. Such a policy will be referred to as an *optimal climate policy*. The properties of equilibria stated in Proposition 1 and the separation result from Section 3 allow us to derive the optimal policy in two steps. In the first step, an efficient climate tax policy τ^{eff} is computed which implements the efficient allocation (52) as an aggregate equilibrium allocation defined as in (45), i.e., $\bar{\mathbf{A}}^*(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$. In the second step, an optimal transfer scheme $\theta^{\text{opt}}(\omega)$ is computed based on a given weighting scheme ω which together with τ^{eff} implements the ω-optimal allocation as an equilibrium allocation, i.e., $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{opt}}(\omega)) = \mathbf{A}^{\text{opt}}(\omega)$. Evidently, the latter requires the choice of a weighting scheme ω. We offer a simple idea how such a scheme could be chosen such that each region has an incentive to adopt the optimal climate tax policy.

4.1. The optimal climate tax policy

Using the efficient allocation (52), define the climate tax policy $\tau^{\text{eff}} = (\tau_t^{\text{eff}})_{t\geq 0}$ as

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n}^{\text{eff}})}{u'(\bar{C}_t^{\text{eff}})} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\Big) \times \sum_{\ell\in\mathbb{L}} D^{\ell'}(S_{t+n}^{\text{eff}}) Q_{0,t}^{\ell} F_0\big(K_{0,t}^{\ell,\text{eff}}, N_{0,t}^{\ell,\text{eff}}, (E_{i,t}^{\ell,\text{eff}})_{i\in\mathbb{I}}\big). \tag{64}$$

Equation (64) is a classical example of a *Pigovian tax* which equates taxes to the total discounted marginal damage caused by each unit of CO_2 determined by (56). Under this policy, emissions taxes $\tau_t^{\ell} \equiv \tau_t^{\text{eff}}$ are uniform across dirty sectors in all countries and incorporate the total damage from emitting one unit of CO_2 in period t. The following result shows that this policy implements the efficient solution (52) as an aggregate equilibrium allocation defined as in (45). Recall from Proposition 1 that this allocation is independent of the transfer policy. Thus, the CTP defined by (64) will be referred to as the *efficient tax policy*.

Theorem 2. *Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). Then, the induced aggregate equilibrium allocation defined in (45) is efficient, i.e., $\bar{\mathbf{A}}^*(\tau^{\text{eff}}) = \bar{\mathbf{A}}^{\text{eff}}$.*

With the specific functional forms (9) and (11) for climate damage and consumer utility, the efficient tax formula (64) takes the more specific form

$$\tau_t^{\text{eff}} = \sum_{n=0}^{\infty} \beta^n \left(\frac{\bar{C}_{t+n}^{\text{eff}}}{\bar{C}_t^{\text{eff}}}\right)^{-\sigma} \Big(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\Big) \sum_{\ell\in\mathbb{L}} \gamma^{\ell} Y_{t+n}^{\ell,\text{eff}}. \tag{65}$$

In particular, efficient taxes are zero if climate damages are absent, i.e., $\gamma^{\ell} \equiv 0$.

In general, expression (65) can not be computed explicitly, as it involves the entire paths of future output in each region and aggregate consumption. However, if the efficient solution induces a balanced growth path on which output and consumption grow at constant and identical rate $g \geq 0$, (65) takes the much simpler form

$$\tau_t^{\text{eff}} = \bar{\tau}^{\text{eff}} \sum_{\ell\in\mathbb{L}} \gamma^{\ell} Y_t^{\ell,\text{eff}}, \quad \bar{\tau}^{\text{eff}} := \frac{\phi_L}{1-\beta(1+g)^{1-\sigma}} + \phi_0 \frac{1-\phi_L}{1-\beta(1+g)^{1-\sigma}(1-\phi)}. \tag{66}$$

Thus, on a balanced growth path, the optimal tax is a constant share $\bar{\tau}^{\text{eff}}$ of world output weighted by the damage parameters γ^{ℓ}. For logarithmic utility ($\sigma = 1$) and homogeneous climate damages ($\gamma^{\ell} \equiv \gamma$), equation (66) recovers precisely the tax-formula derived in GHKT under a set of additional restrictions (log utility, Cobb-Douglas production, all capital used in the final sector).[17] None of these restrictions is required here if the assumption of a balanced growth path is satisfied. The numerical simulations presented in Hillebrand and Hillebrand (2018) and in Section 5 below show that the efficient solution converges quickly to a balanced growth path suggesting that (65) is well-approximated by (66) in applications of the model.

4.2. Optimal transfer policies

Under the efficient climate tax policy (64), the aggregate equilibrium allocation (45) coincides with the efficient allocation (52). While this determines aggregate world consumption together with an optimal allocation of production factors and climate variables in each period, it leaves undetermined how world consumption is distributed across countries. The latter requires the choice of a weighting scheme $\omega \in \Delta_+$ based on which the optimal consumption distribution $\mu^{\text{opt}}(\omega)$ can be determined by (53).

We now explore the existence of a transfer policy θ under which the induced equilibrium allocation $\mathbf{A}^*(\tau^{\text{eff}}, \theta)$ coincides with the optimal allocation $\mathbf{A}^{\text{opt}}(\omega)$ defined in (49) in which each region attains the optimal consumption share $\mu^{\text{opt}}(\omega)$. As before, let τ^{eff} be the efficient tax policy which implements the efficient aggregate allocation $\bar{\mathbf{A}}^{\text{eff}}$ and, by Lemma 3 also determines the price system $\mathbf{P}^{\text{eff}}$ supporting the efficient allocation. Let $W^{\ell,\text{eff}}$ denote the induced lifetime non-transfer income of consumers in region ℓ defined in (31), $W^{\text{eff}} := \sum_{\ell \in \mathbb{L}} W^{\ell,\text{eff}}$ aggregate non-transfer lifetime income and T^{eff} the total tax revenue defined as in (15). Given a weighting scheme $\omega \in \Delta_+$ and consumption shares $\mu^{\text{opt}}(\omega) = (\mu^{\ell,\text{opt}}(\omega))_{\ell \in \mathbb{L}}$ determined by Lemma 4, consider the following transfer policy $\theta^{\text{opt}} = (\theta^{\ell,\text{opt}})_{\ell \in \mathbb{L}}$ defined for each $\ell \in \mathbb{L}$ as

$$\theta^{\ell,\text{opt}}(\omega) = \frac{\mu^{\ell,\text{opt}}(\omega)\left(W^{\text{eff}} + T^{\text{eff}}\right) - W^{\ell,\text{eff}}}{T^{\text{eff}}}. \tag{67}$$

Note that (67) determines consumer ℓ's lifetime cum-transfer income $W^{\ell} + T^{\ell}$ to be a share $\mu^{\ell,\text{opt}}(\omega)$ of world cum-transfer income $W^{\text{eff}} + T^{\text{eff}}$. The following result shows that the transfer policy (67) together with τ^{eff} constitutes indeed an optimal climate policy.

Theorem 3. *Let Assumptions 1 and 2 hold and define the climate tax policy τ^{eff} as in (64). Given any weighting scheme $\omega \in \Delta_+$, define $\mu^{\text{opt}}(\omega)$ by (53) and the transfer policy $\theta^{\text{opt}}(\omega)$ by (67). Then, the induced equilibrium allocation is ω-optimal, i.e., $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{opt}}(\omega)) = \mathbf{A}^{\text{opt}}(\omega)$.*

4.3. A Pareto-improving transfer policy

Applying the optimal transfer policy defined in (67) requires the choice of a particular weighting scheme $\omega \in \Delta_+$ or, equivalently, invoking Lemma 4 (ii), a desired consumption distribution $\mu = (\mu^{\ell})_{\ell \in \mathbb{L}} \in \Delta_+$. This raises the question how such a distribution can and should be determined. Ideally, one might want to choose ω resp. μ as an equal weighting scheme based, e.g.,

[17] A general formula for the Social Cost of Carbon and its approximation along a balanced growth path is derived in van den van den Bijgaart et al. (2016).

on relative population sizes to ensure a fair world allocation of consumption. In any quantitative study of the model, however, such a choice would induce massive transfers unrelated to climate change but due to the fact that the world income distribution is very unequal and strongly biased towards industrialized countries.

The analysis of this paper, however, is not about fairness and redistribution of world income but how transfers can be determined such that each region has an incentive to implement the optimal tax on emissions. For this reason, the present section offers an alternative approach which chooses the consumption distribution based on the shares that each region attains in the Laissez faire allocation. This target seems a natural choice because the Laissez faire solution corresponds to the extreme case where all countries agree not to take any measures against climate change. It is therefore a natural threat point in any bargaining process about transfers.

Formally, let $\mu^{\mathrm{LF}} = (\mu^{\ell,\mathrm{LF}})_{\ell \in \mathbb{L}}$ denote the consumption shares along the Laissez faire equilibrium allocation $\mathbf{A}^{\mathrm{LF}}$ which are constant by Lemma 3 (ii). Using the same notation as in the previous subsection, define the transfer policy $\theta^{\mathrm{LF}} = (\theta^{\ell,\mathrm{LF}})_{\ell \in \mathbb{L}} \in \Delta$ as

$$\theta^{\ell,\mathrm{LF}} := \frac{\mu^{\ell,\mathrm{LF}}\left(W^{\mathrm{eff}} + T^{\mathrm{eff}}\right) - W^{\ell,\mathrm{eff}}}{T^{\mathrm{eff}}}, \quad \ell \in \mathbb{L}. \tag{68}$$

Under transfer policy θ^{LF}, each region ℓ attains the same relative wealth and the same share $\mu^{\ell,\mathrm{LF}}$ of world consumption along the efficient equilibrium allocation as in the Laissez faire allocation. The following main result shows that this policy makes each country better-off, i.e., consumers in each region enjoy utility strictly higher than in the Laissez faire allocation if they agree to jointly implement the efficient tax policy.[18]

Theorem 4. *The equilibrium allocation $\mathbf{A}^*(\tau^{\mathrm{eff}}, \theta^{\mathrm{LF}})$ Pareto-improves the laissez faire allocation $\mathbf{A}^{\mathrm{LF}}$.*

5. Quantitative results

This section provides a quantitative example which illustrates the previous theoretical results.[19] We consider the case with two regions ($L = 2$) which are broadly calibrated to match rich ($\ell = 1$) and poor ($\ell = 2$) world regions represented by OECD and Non-OECD countries. Details on these calibrations and the computation of equilibria in our model can be found in Hillebrand and Hillebrand (2018).

5.1. Calibration

The simulation starts in $t = 2015$ and ends in $t = 2215$ with one period representing ten years which is a standard choice in the literature. Initial world labor supply $N_0^{1,s} + N_0^{2,s}$ is normalized to one and distributed across regions based on empirical population shares. Regional differences in productivity are captured by the parameters $Q_{0,t}^{\ell} \equiv Q_0^{\ell}$ in (1) which are constant and chosen to match the world income distribution between regions and an initial world output of 700 trillion current US$ as in GHKT. Growth enters our model via labor augmenting technological change

[18] Clearly, this does not eliminate the free-riding problem that a single region may have an incentive to deviate from the optimal policy. A more elaborate game-theoretic analysis of this problem within the previous framework is beyond the scope of this paper but left for future research.

[19] The simulation data are available at http://www.marten-hillebrand.de/research.

Table 1
Parameter set.

Consumer sector		Final sector		Energy sector 1		Energy sector 2		Climate model	
$N_0^{1,s}$	0.18	Q_0^1	3.23	Q_1^1	4.2	Q_2^1	20	$S_{1,-1}$	722
$N_0^{2,s}$	0.82	Q_0^2	0.65	Q_1^2	12	Q_2^2	40	$S_{2,-1}$	110
K_0	0.18	α_0	0.3	α_1	0.52	α_2	0.75	ϕ_L	0.2
g	0.16	ν_0	0.04	ν_1	0.27			ϕ	0.0228
β	0.985^{10}	ϱ	3	c_1	0.000071			ϕ_0	0.393
σ	1	κ	0.5	ζ_1	0.5835			γ^ℓ	0.000053
								$\bar{S}$	581

due to which labor supply $N_t^{\ell,s}$ in each region ℓ grows at constant rate g. As argued in the next section, population growth can be incorporated in the discount factor β which is chosen as in GHKT. We also assume a logarithmic utility function by setting $\sigma = 1$.

There are two energy sectors ($I = 2$) one of which ($i = 1$) produces dirty energy based on fossil fuels and the other ($i = 2$) clean energy based on renewable resources. The production functions in equations (1), (2), and (3) are specified as follows:

$$F_0(K, N, E^1, E^2) = K^{\alpha_0} N^{1-\alpha_0-\nu_0}[\kappa(E^1)^{\frac{\varrho-1}{\varrho}} + (1-\kappa)(E^2)^{\frac{\varrho-1}{\varrho}}]^{\frac{\varrho\nu_0}{\varrho-1}} \tag{69a}$$

$$F_1(K, N, X) = X^{\nu_1} K^{\alpha_1} N^{1-\alpha_1-\nu_1}, \; F_2(K, N) = K^{\alpha_2} N^{1-\alpha_2}. \tag{69b}$$

We choose the shares of capital α_0 and energy ν_0 in (69a) as in GHKT and set $\varrho = 3$ to obtain a high substitutability between clean and dirty energy as in Acemoglu et al. (2012) or Rezai and van der Ploeg (2015) who even assume the two energy types to be perfect substitutes. Choosing $\kappa = 0.5$ yields a unit relative price between clean and dirty energy also assumed in GHKT. The production elasticities $\alpha_1, \nu_1, \alpha_2$ in (69b) are chosen to match the empirical cost structure of energy sectors reported in Hillebrand and Hillebrand (2018). Productivity parameters $Q_{i,t}^\ell \equiv Q_i^\ell$ in energy production (2) and (3) are constant and chosen to obtain an empirically plausible energy mix in each region with clean energy acquiring a share of 22% in rich and 14% in poor countries in the initial period (see Hillebrand and Hillebrand, 2018 for details).

Exhaustible resources in our simulation model comprise all fossil fuels (coal, oil, and natural gas). As GHKT, we assume that these resources are abundant and do not have a scarcity rent by setting $R_0^1 = \infty$. By (24) and Lemma 3(a), resource prices are thus equal to extraction costs c_1 in each period. To obtain a plausible value for c_1, we combine the two estimates for coal and oil extraction from Hillebrand and Hillebrand (2018) weighted by the empirical global resource shares. The same approach determines the Carbon content ζ_1 of fossil fuels as a weighted average of coal, oil, and natural gas.

The parameters defining the climate model (6) and damage function (9) are chosen as in GHKT and the initial climate state $\mathbf{S}_{-1} = (S_{1,-1}, S_{2,-1})$ is chosen to match the empirically observed CO_2 concentration of 850 Gigatons of carbon in $t = 2015$ with the same shares of permanent and non-permanent carbon as in GHKT. Table 1 summarizes our parameter set.

5.2. *Equilibrium dynamics*

Our parametrization yields an optimal carbon tax equal to 34 \$/t CO_2 in 2015, which is in the range of optimal emissions taxes reported, e.g., in Nordhaus (2007) and Golosov et al. (2014).

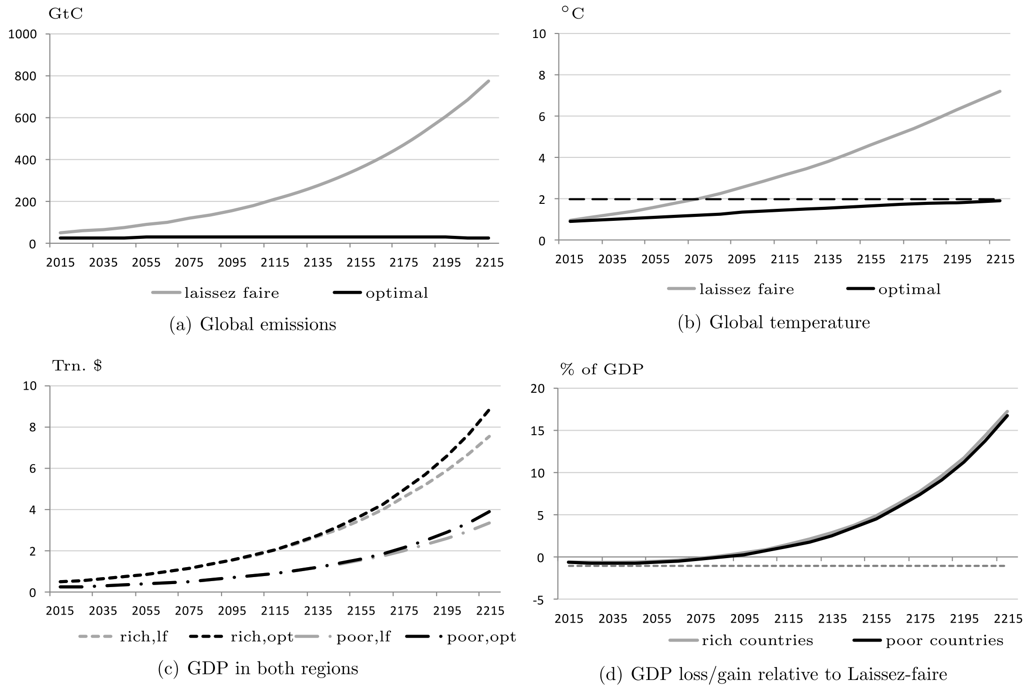

Fig. 1. Evolution under laissez faire and optimal taxation.

Comparing the Laissez-faire and optimal solution, Fig. 1 depicts the predicted evolution of selected economic and climate variables over the next 200 years. As one would expect, the optimal emissions tax curbs global emissions which stay flat over the entire time window and decline to zero asymptotically. This limits the increase in global temperature[20] to a maximum of 2 degrees relative to pre-industrial level and is thus in line with the two-degree target. Laissez-faire causes an exponential increase in emissions and temperature exceeding the two-degree target within the next 50 years. At the disaggregated level, GDP in both regions follows a balanced growth path under optimal taxation with roughly constant growth rates while these growth rates decline continually under Laissez-faire. Introducing the optimal tax comes at a slight initial cost which is limited to at most 0.8% of GDP in both regions. After fifty years, the gain in GDP relative to Laissez faire becomes positive and increases continually thereafter.

5.3. *Pareto-improving transfers*

The distribution of global tax revenue depends on the weighting scheme $\omega = (\omega^1, \omega^2)$ which is fully determined by the weight ω^1 attached to rich countries. By Lemma 4 and (54), our choice $\sigma = 1$ permits to interpret ω^1 directly as the target consumption share of rich countries along the optimal equilibrium. An important reference is the case where ω^1 equals the consumption share along the laissez faire equilibrium. Denote this choice by ω^*. By Theorem 3, choosing $\omega^1 = \omega^*$

[20] Following GHKT, we compute global temperature in period t as $TEMP_t = \lambda \log(S_t/\bar{S})/\log(2)$, cf. their equation (25) on page 66.

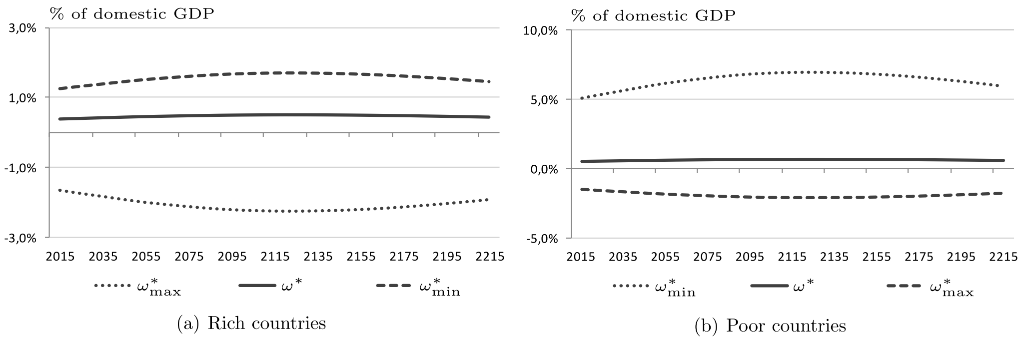

Fig. 2. Pareto-improving transfers between rich and poor countries.

induces a Pareto improvement relative to laissez faire. To obtain the set of Pareto-improving weighting schemes, we compute two critical values $\omega^*_{\min}$ and $\omega^*_{\max}$. For $\omega^1 = \omega^*_{\max}$, region 2 is exactly indifferent between the optimal equilibrium and laissez faire while region 1 is exactly indifferent if $\omega^1 = \omega^*_{\min}$. Thus, any choice $\omega^1 \in [\omega^*_{\min}, \omega^*_{\max}]$ induces a Pareto-improvement. For each of the three cases $\omega^1 \in \{\omega^*_{\min}, \omega^*, \omega^*_{\max}\}$, Fig. 2 depicts the associated transfers received by each region which are expressed as a percentage of regional GDP. For $\omega^1 = \omega^*$, rich countries are entitled to receive about 63.4% of global tax revenue corresponding to a transfer of roughly half a percentage point of their GDP in each period. Transfers received by poor countries vary between 0.5% and 0.7% of domestic GDP. For $\omega^1 = \omega^*_{\max}$, rich countries receive 213.8% of global tax revenue which must partially be financed by consumers in the poor region who would have to pay a lump-sum tax. This tax amounts to 1.5% of GDP in the initial period and increase to up 2.1% in subsequent periods. Conversely, for $\omega^1 = \omega^*_{\min}$, poor countries receive a share of 382.7% of total tax revenue in which case rich countries must levy a tax on their consumers. This tax amounts to initially 1.6% of GDP and increases to up to 2.3% in subsequent periods. These figures define a range of possible transfer payments that both regions could agree on in negotiations on the optimal climate policy.

In absolute terms, transfers in the initial period compute as follows. Annual GDP in rich countries is 48.8 trillion U.S.$ and 21.4 trillion U.S.$ in poor countries. Global annual tax revenue is about 283.7 billion U.S.$. For $\omega^1 = \omega^*$, rich countries receive an annual transfer of about 179.9 billion U.S.$ and poor countries about 103.8 billion U.S.$. If $\omega^1 = \omega^*_{\max}$, poor countries must raise an annual tax revenue of 322.8 billion U.S.$ to finance an annual transfer of 606.5 billion U.S.$ to rich countries. Finally, if $\omega^1 = \omega^*_{\min}$, poor countries receive an annual transfer of 1085.6 billion U.S.$ part of which is financed by a tax on consumers in rich countries which amounts to 801.6 billion U.S.$.

6. Extensions and discussion

The present section discusses which of our previous restrictions on technologies, preferences, and the climate model are crucial for our results and which ones are merely convenient to work with and can be relaxed. For the sake of readability, some of the underlying formal arguments are relegated to Section A.11 in the appendix.

6.1. Climate change affecting utility

An alternative approach to modeling the adverse effects of climate change is to assume a direct impact on consumer utility. Models which adopt this idea include Acemoglu et al. (2012), Barrage (2017), Gerlagh and Liski (2016, 2018), or Rezai and van der Ploeg (2016). Qualitatively, all our main results continue to hold if we modify our period utility function (11) to be of the form

$$u(C,S)=\log C-v(S) \tag{70}$$

where v is any differentiable, increasing, and concave function of carbon concentration defined as in (7). Since both climate damage D_t and global temperature can be written as functions of S_t, specification (70) also includes cases where utility depends negatively on global temperature or damage as in Barrage (2017).

In the decentralized solution, consumers in each region now take the sequence $(S_t)_{t\geq 0}$ as an additional exogenous parameter in their decision problem. The additive structure (70) then implies that the solution to this problem and, therefore, all equilibrium conditions derived in Section 2 remain unchanged. By contrast, both the weighted and aggregate planning problems (48) and (51) now incorporate the direct impact of emissions on utility. Thus, the social costs of carbon now comprises both the damage to production and consumer utility. Formally, $\Lambda_t = \Lambda_t^{\text{prod}} + \Lambda_t^{\text{cons}}$ where Λ_t^{prod} is determined by (56) as before while Λ_t^{cons} captures the marginal disutility from emissions given by

$$\Lambda_t^{\text{cons}} = \sum_{n=0}^{\infty} \beta^n \frac{\bar{C}_{t+n}^{-1}}{\bar{C}_t^{-1}} \left(\phi_L + (1-\phi_L)\phi_0(1-\phi)^n\right) \frac{v'(S_{t+n})}{\bar{C}_{t+n}^{-1}}. \tag{71}$$

Note the similarity of the previous formula to the one derived in Barrage (2017), see her equation (20). Economically, Λ_t^{cons} represents the marginal welfare loss of a fictitious representative world consumer who has the same utility (70) as households in each region. In this regard, it is important to note that the separability result from Theorem 1 requires utility (70) to be logarithmic in consumption and breaks down under the more general form (11) if $\sigma \neq 1$. Defining the emissions tax as $\tau_t^{\text{eff}} = \Lambda_t$ for all t then implements the efficient solution as an aggregate equilibrium allocation as before.

6.2. An alternative climate model

Our specification of the climate model is taken directly from GHKT. An alternative, more recent specification is developed in Gerlagh and Liski (2018) which permits to decompose the size, delay, and persistence of emissions in the atmosphere. Using our notation, the key equation of their climate model determines atmospheric CO_2 as

$$S_t = \sum_{\tau=0}^{\infty} \delta_\tau Z_{t-\tau}. \tag{72}$$

Here, Z_t denotes total CO_2-emissions defined as in (5) which are set to zero for $t \leq T$ where T is the beginning of the industrial revolution. Equation (72) is derived in Gerlagh and Liski (2018) based on a deeper specification of the underlying climate model with multiple layers which contains the three-layer specification used in the DICE/RICE model as a special case. Thus, equation

(72) also covers the climate model adopted in Nordhaus and Boyer (2000). Also note that Gerlagh and Liski (2018) assume $\delta_0 = 0$ in which case emissions do not have an instantaneous effect on the climate state.

Using now (5), (8), and (72) as our climate model while maintaining all other previous assumptions, the main results of the previous analysis continue to hold. In particular, the efficient allocation can be computed exactly as before and permits to obtain an optimal allocation via Theorem 3. The only major change is that the social cost of carbon (56) defining the efficient tax τ_t^{eff} (64) now takes the form

$$\Lambda_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \delta_n \sum_{\ell \in \mathbb{L}} D^{\ell'}(S_{t+n}) Q^{\ell}_{0,t+n} F_0\big(K^{\ell}_{0,t+n}, N^{\ell}_{0,t+n}, (E^{\ell}_{i,t+n})_{i\in\mathbb{I}}\big). \tag{73}$$

As Gerlagh and Liski (2018) also assume an exponential damage function, their model is included by our damage function (9) with region-specific damage parameters γ^{ℓ}.[21] Under this specification, the social cost of carbon (73) takes the simpler form

$$\Lambda_t = \sum_{n=0}^{\infty} \beta^n \frac{u'(\bar{C}_{t+n})}{u'(\bar{C}_t)} \delta_n \sum_{\ell \in \mathbb{L}} \gamma^{\ell} Y^{\ell}_{t+n}. \tag{74}$$

One observes that due to (11) the SCC (74) is again proportional to damage weighted output $\sum_{\ell\in\mathbb{L}} \gamma^{\ell} Y^{\ell}_t$ on a balanced growth path. Thus, the alternative climate model (72) fully preserves our multi-region version (66) of the GHKT result.

6.3. Population growth

The previous framework formally assumed a stationary population in each region. It is now straightforward to modify this restriction and include constant population growth in our model. To see this, denote total consumption in region ℓ by C^{ℓ}_t as before and let L^{ℓ}_t denote the population size corresponding to the number of consumers in region ℓ in period t. Assume that the population grows at constant rate g^{ℓ}_L such that

$$L^{\ell}_t = (1 + g^{\ell}_L) L^{\ell}_{t-1} \tag{75}$$

with the initial population to normalized to $L^{\ell}_0 = 1$. Adopting the same approach as in Rezai and van der Ploeg (2015), define aggregate utility in region ℓ as

$$U((C^{\ell}_t)_{t\geq 0}) = \sum_{t=0}^{\infty} L^{\ell}_t \beta^t u(C^{\ell}_t / L^{\ell}_t) \tag{76}$$

which replaces our earlier specification (10). Exploiting the functional form (11) and using $L^{\ell}_0 = 1$ and (75) permits to write utility (76) equivalently as

$$U((C^{\ell}_t)_{t\geq 0}) = \sum_{t=0}^{\infty} (\beta(1 + g^{\ell}_L))^t u(C^{\ell}_t) + \bar{u}$$

[21] In the notation of Gerlagh and Liski (2018), final output can be expressed as $y_t = A_t \omega(s_t) f(k_t, e_t, l_{y,t})$ where total factor productivity is $\omega(s_t) = \exp\{-\sum_{\tau=1}^{\infty} \theta_\tau z_{t-\tau}\}$ with s_t being the history of emissions before t. Function f depends on capital, energy, and labor and A_t is a productivity parameter.

if $\sigma = 1$ and

$$U((C_t^\ell)_{t\geq 0}) = \sum_{t=0}^{\infty}(\beta(1+g_L^\ell)^\sigma)^t u(C_t^\ell) + \bar{u}$$

if $\sigma \neq 1$. Here, $\bar{u}$ is a constant determined by exogenous parameters that has no economic implications. Thus, population growth merely transforms the original discount factor β to $\hat{\beta} := \beta(1+g_L^\ell)^\sigma$. Since our results require all regions to share the same discount factor, the model is compatible with constant population growth as long as growth rates are uniform across regions, i.e., $g_L^\ell \equiv g_L$ and the transformed discount factor satisfies $\hat{\beta} < 1$. In this case, all our previous results continue to hold without change.

6.4. Redistribution via non-lump sum transfers

The revenue from taxation of emissions is distributed to consumers in the form of lump-sum transfers. In the literature, the assumption of lump-sum taxes is frequently challenged as unrealistic. In principle, similar arguments could be made against lump-sum transfers, although they seem much less controversial.[22] Thus, it is important to discuss how our results change if lump-sum transfers are not available.

Alternative transfers could take the form of proportional subsidies on labor income, capital income, or final consumption. Denoting the percentage subsidy on these incomes as $\eta_{n,t}^\ell$, $\eta_{k,t}^\ell$ $\eta_{c,t}^\ell$, respectively, the consumer's period budget constraint (28) changes to

$$C_t^\ell(1-\eta_{c,t}^\ell) + K_{t+1}^\ell = (1+\eta_{k,t}^\ell)r_t K_t^\ell + (1+\eta_{n,t}^\ell)w_t^\ell N_t^{\ell,s} + \Pi_t^\ell. \tag{77}$$

As before, denote by θ^ℓ the share of total tax revenue $T_t := \tau_t \sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\zeta_i X_{i,t}^\ell$ received by region ℓ. These revenues are fully distributed as subsidies to consumers such that

$$\eta_{c,t}^\ell C_t^\ell + \eta_{k,t}^\ell r_t K_t^\ell + \eta_{n,t}^\ell w_t^\ell N_t^{\ell,s} = \theta^\ell T_t. \tag{78}$$

The consumer's intertemporal optimality condition (34) now takes the form

$$r_{t+1}(1+\eta_{k,t+1}^\ell)\beta\left(\frac{C_{t+1}^\ell}{C_t^\ell}\right)^{-\sigma}\frac{1-\eta_{c,t}^\ell}{1-\eta_{c,t+1}^\ell} = 1. \tag{79}$$

Note that wage subsidies do not enter this condition. Thus, if all transfers are paid as proportional subsidies $\eta_{n,t}^\ell$ on labor income, this merely changes the consumers lifetime income (31) to $W^\ell = r_0 K_0^\ell + \Pi^\ell + \sum_{t=0}^\infty q_t(1+\eta_{n,t}^\ell)w_t^\ell N_t^{\ell,s}$. Thus, the structural properties of equilibrium from Proposition 1 remain unchanged and by setting $\eta_{n,t}^\ell := \frac{\theta^\ell T_t}{w_t^\ell N_t^{\ell,s}}$, an optimal climate policy can be obtained exactly as before. Clearly, the key feature required for this result is that labor supply is exogenous which is a standard assumption made in most models of climate change (an

[22] The assumption of lump sum transfers is also made in GHKT and in Hassler and Krusell (2012). Even Barrage (2017), who studies how distortionary taxation affects the optimal climate tax assumes that governments can make lump-sum transfers to consumers.

exception is Barrage (2017)).[23] In this case, wage subsidies are non-distortionary and essentially equivalent to lump sum transfers.

One also observes from (79) that the distortionary effect of consumption subsidies vanishes if $\eta^{\ell}_{c,t} \equiv \eta^{\ell}_{c}$. Such a constant choice, however, will typically not satisfy condition (78) for all t. If constant consumption subsidies are amended by non-distortionary wage subsidies chosen to satisfy (78), Proposition 1 and all major results remain true.

Transfers paid either as capital subsidies or time varying consumption subsidies distort the consumer's intertemporal first order condition. In such cases, aggregation of this constraint, which is key for the result from Proposition 1 is no longer possible. While the first best optimal allocation could still be determined as before, implementing this allocation as an equilibrium by separating efficiency from distribution is no longer possible, since transfers also affect the aggregate equilibrium allocation. To determine an optimal climate policy based on distortionary transfers, one would most likely have to employ a Ramsey framework as in Barrage (2017) which incorporates the response of consumers to the proposed climate policy and maximizes a weighted utility index of the form (47). We do not expect to preserve our separation result under such a modification. In this sense, the existence of non-distortionary transfers is crucial for our main results.

6.5. Relation to RICE and Negishi weights

It is important to contrast our approach of determining an optimal policy to the one adopted in the RICE model as described in Nordhaus and Yang (1996), see also Nordhaus and Boyer (2000). To determine an efficient allocation in this model, all endogenous variables are determined by solving a planning problem of the type (48) with (initially) constants weights. Unlike our approach which treats these weights as policy parameters, however, the RICE algorithm determines the weights endogenously to make the solution compatible with an (asymptotic) no-trade equilibrium. This is required by the restrictions on regional trade in the RICE framework. While the existence of such weights would ensure that the resulting allocation is Pareto-optimal and, therefore, can be interpreted as a competitive equilibrium allocation due to Negishi's theorem (Negishi, 1960), the solution is then again perturbed by choosing time-varying weights, referred to as 'modified Negishi-weights'. This perturbation is done in order to avoid excessive reallocation of capital across regions but does not necessarily lead to a Pareto-optimal allocation (see Denning and Emmerling, 2017 for a discussion of these problems). Thus, the analysis in RICE does not determine a socially optimal allocation in our sense for arbitrary weights but the solution is – at best – Pareto optimal. Further, it is not possible to treat the welfare weights as policy parameters representing bargaining power, population size, fairness considerations, etc. as we do. Instead, they are dictated by economic restrictions and the employed methodology in the RICE framework.

[23] From an empirical standpoint, the assumption of perfectly inelastic labor supply seems justified whenever the length of each time period is sufficiently long such that short term business cycle fluctuations are eliminated. This is certainly the case if one model period corresponds to ten years, a standard parametrization used in Acemoglu et al. (2012), GHKT, and also in Section 5.

6.6. Relation to single-region models

Disaggregated consumption in our model is fully described by aggregate consumption. This raises the question whether the production side can be aggregated in a similar way such that world output can be written as a function of aggregate factor inputs. It turns out that the answer is negative unless technologies in all regions are identical. To see this formally, consider an arbitrary period t. Dropping time index t for convenience, let damages D^ℓ and sector-specific productivities $(Q_i^\ell)_{i\in\mathbb{I}_0}$ in each region $\ell \in \mathbb{L}$ be given. Substitute (2) and (3) into (1) to obtain final output in region ℓ as

$$Y^\ell = F^\ell((K_i^\ell, N_i^\ell, X_i^\ell)_{i\in\mathbb{I}_0}) := (1 - D^\ell)Q_0^\ell F_0(K_0^\ell, N_0^\ell, (Q_i^\ell F_i(K_i^\ell, N_i^\ell, X_i^\ell))_{i\in\mathbb{I}}).$$

The derived production functions F^ℓ are linear-homogeneous. They are identical across regions precisely when damages and productivities are identical, i.e., $F^\ell \equiv F$ if and only if $D^\ell \equiv D$ and $Q_i^\ell \equiv Q_i$ for all $i \in \mathbb{I}_0$. In this case, standard arguments can be used to show that aggregate output $Y := \sum_{\ell\in\mathbb{L}} Y^\ell$ is determined by aggregate inputs of capital $K_i := \sum_{\ell\in\mathbb{L}} K_i^\ell$, labor $N_i := \sum_{\ell\in\mathbb{L}} N_i^\ell$, and exhaustible resources $X_i := \sum_{\ell\in\mathbb{L}} X_i^\ell$, i.e.,

$$Y = F((K_i, N_i, X_i)_{i\in\mathbb{I}_0}).$$

The previous argument breaks down as soon as climate damages or productivities in at least one sector differ across regions. Thus, it is not possible to represent the production side by an aggregate technology except for the trivial case where regions are essentially identical. In all other cases, world output is determined by a technology that involves all disaggregated production variables. In our calibrated example and also in our companion paper Hillebrand and Hillebrand (2018), we show that differences in productivities are crucial to match certain regional characteristics of the production process. Moreover, differences in climate damages are known to be particularly important when regions represent rich and poor countries, cf. Bretschger and Suphaphiphat (2014) and references therein. For these reasons, our framework is not equivalent to a single region model but permits to study many important questions which are due to regional heterogeneity and can not be addressed in a single region model.

7. Conclusions

The problem of determining an optimal climate path and an efficient allocation of production factors and exhaustible resources admits a unique solution which is completely independent of the interests of different countries. This solution can be implemented as a decentralized equilibrium by levying a uniform global tax on carbon emission which can be computed (or approximated) in closed form. In principle, all countries should agree on this policy.

The real issue in the political debate about climate change is therefore not how and where emissions should be taxed, but rather how countries should share the tax revenue via transfers. These transfers determine the world distribution of consumption or income and provide a mechanism to compensate regions for climate damages. As the choice of an optimal transfer policy induces a trade-off between the interests of different countries, one might want to determine the transfer policy such that each region has an incentive to implement the optimal tax policy. An example of a transfer policy which leads to a Pareto-improvement relative to the Laissez faire allocation was devised in this paper.

The latter results mark only a first step towards a more elaborate model of the political process which determines climate policies. In future research, we intend to model this process as a (co-operative or non-cooperative) game between regions as in Dutta and Radner (2006). Within the framework of this paper, $\mathbb{L}$ would be the set of players each of which chooses a domestic emissions tax policy τ^{ℓ} as their strategy and receives utility of domestic consumers as their pay-off. Transfers across regions then correspond to side payments which can be used to incentivize each region to implement a certain strategy. This raises the question whether there exists a transfer policy under which the optimal climate tax policy derived in this paper can be obtained as the Nash equilibrium of this non-cooperative game. One could also study cooperative versions of this game where some regions join forces to combat climate change by forming coalitions.

In addition, the framework developed in this paper can be extended in various directions. One such extension is to replace the deterministic setup by a stochastic environment with random perturbations which allows to include various forms of uncertainty into the model. A second extension is a setup with endogenous growth and directed technical change as in Acemoglu et al. (2012). Both modifications were considered in GHKT and we believe that our framework is also amendable to them.

It is also well-known in the literature that a uniform carbon tax across regions may fail to be optimal if lump-sum transfers are not available, cf. Chichilnisky and Heal (1994) or d'Autume et al. (2016) and also the discussion in Section 6.4. Including such additional restrictions in our model to study how this affects the optimal climate policy is a final goal of future research.

Appendix A. Mathematical Appendix

A.1. Proof of Lemma 1

Using standard Lagrangian arguments, a non-negative sequence $(X_t^*)_{t\geq 0}$ is a solution to (23) if $\sum_{t=0}^{\infty} X_t^* \leq R_{i,0}^{\ell}$ and there exist non-negative Lagrangian variables $(\sigma_t^*)_{t\geq 0}$ and $\lambda \geq 0$ such that $((X_t^*, \sigma_t^*)_{t\geq 0}, \lambda^*)$ solve the first order and Kuhn-Tucker conditions

$$q_t(v_{i,t} - c_i) + \sigma_t - \lambda = 0 \forall t \geq 0 \tag{A.1a}$$

$$\sigma_t X_t = 0 \forall t \geq 0 \tag{A.1b}$$

$$\lambda\Big(\sum_{t=0}^{\infty} X_t - R_{i,0}^{\ell}\Big) = 0. \tag{A.1c}$$

If $X_t^* > 0$ for all $t \geq 0$, then, $\sigma_t^* = 0$ by (A.1b) and $v_{i,t} \geq c_i$ by (A.1a) for all t. Using $q_0 = 1$ and $q_t/q_{t-1} = r_t^{-1}$ for all $t > 0$ in (A.1a), resource prices must evolve as in (24). The remaining assertions follow immediately. □

A.2. Proof of Lemma 2

Under Assumption 2, (12) and (34) imply that the solution to (32) evolves as

$$C_t^{\ell *} = C_{t-1}^{\ell *}(\beta r_t)^{\frac{1}{\sigma}} = C_0^{\ell *}\prod_{s=1}^{t}(\beta r_s)^{\frac{1}{\sigma}} = C_0^{\ell *}\left(\frac{\beta^t}{q_t}\right)^{\frac{1}{\sigma}} \qquad t \geq 1. \tag{A.2}$$

Using (A.2), the l.h.s. in the lifetime budget constraint (30) can be written as

$$\sum_{t=0}^{\infty} q_t C_t^{\ell *} = \sum_{t=0}^{\infty} q_t C_0^{\ell *} \left(\beta^t / q_t\right)^{\frac{1}{\sigma}} = C_0^{\ell *} \sum_{t=0}^{\infty} \left(\beta^t q_t^{\sigma - 1}\right)^{\frac{1}{\sigma}}. \tag{A.3}$$

Using (A.3) in (30) – which holds with equality – gives

$$C_0^{\ell *} = \frac{W^\ell + T^\ell}{\sum_{t=0}^{\infty} \left(\beta^t q_t^{\sigma - 1}\right)^{\frac{1}{\sigma}}}. \tag{A.4}$$

Using (A.4) in (A.2) yields (36). □

A.3. *Proof of Lemma 3*

(i) This is a direct consequence of the equilibrium conditions (18), (20), (22) and the boundary behavior of F_i and u imposed by Assumptions 1 and 2.

(ii) Set $\mu^{\ell *} := C_0^{\ell *} / \bar{C}_0^*$ for $\ell \in \mathbb{L}$. By Lemma 2, the growth rates of each sequence $(C_t^{\ell *})_{t \geq 0}$ are independent of ℓ and equal to the growth rates of aggregate consumption $(\bar{C}_t^*)_{t \geq 0}$. Induction then implies that (44) holds for all $t \geq 0$.

(iii) Let $i \in \mathbb{I}_x$ be arbitrary. If $R_{i,0} = \infty$, there exists a region $\ell \in \mathbb{L}$ for which $R_{i,0}^\ell = \infty$. As profits must be finite at equilibrium, (25) implies $v_{i,0} - c_i$.

If $R_{i,0} < \infty$, (41) implies $\lim_{t \to \infty} X_{i,t}^\ell = 0$ for all $\ell \in \mathbb{L}$. The boundary behavior from Assumption 1 gives $\lim_{t \to \infty} \partial_X F_i\left(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell\right) = \infty$ and the claim therefore follows from (22c).

If $R_{i,0} > \sum_{t=0}^{\infty} \sum_{\ell \in \mathbb{L}} X_{i,t}^{\ell *}$, the same arguments employed to prove (b) give the second result while the first one follows from Lemma 1 (ii). □

A.4. *Proof of Proposition 1*

(i) Using the result from Lemma 2, aggregate consumption in period t satisfies

$$\bar{C}_t^* = \frac{\left(\beta^t / q_t\right)^{\frac{1}{\sigma}} \left[\sum_{\ell \in \mathbb{L}} W^\ell + T\right]}{\sum_{s=0}^{\infty} q_s \left(\beta^s / q_s\right)^{\frac{1}{\sigma}}} \qquad t \geq 0 \tag{A.5}$$

and determines K_{t+1} by (42). These and all other equations relevant to determine the variables in $\bar{\mathbf{A}}^*$ and prices $\mathbf{P}^*$ are independent of θ.

(ii) Using Lemma 2 and (14) in conjunction with (46) and (A.5) gives

$$C_t^{\ell *} = \frac{\left(\beta^t / q_t\right)^{\frac{1}{\sigma}} \left[W^\ell + \theta^\ell T\right]}{\sum_{s=0}^{\infty} q_s \left(\beta^s / q_s\right)^{\frac{1}{\sigma}}} = \mu^{\ell *} \bar{C}_t^*$$

for all $t \geq 0$ and $\ell \in \mathbb{L}$, proving the claim. □

A.5. *Proof of Lemma 4*

(i) Let $\omega \in \Delta_+$ and $\bar{C} > 0$ be given. It is clear that $\omega^\ell = 0$ implies $\mu^\ell = 0$ which is implied by the solution (54). As we can always restate problem (54) as a maximization problem involving only those μ^ℓ for which $\omega^\ell > 0$, the remainder of the proof assumes w.l.o.g. that $\omega \gg 0$. The boundary behavior of u defined in (11) implies that any solution to (53) is bounded away from zero by, say, $\underline{\mu} = (\underline{\mu}^\ell)_{\ell \in \mathbb{L}} \gg 0$. This and the constraint $\sum_{\ell \in \mathbb{L}} \mu^\ell \leq 1$ define a compact, convex

subset of $\mathbb{R}^L_{++}$ on which the map $\mu \mapsto \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell)$ is continuous and strictly concave ensuring that (53) has a unique solution μ^{opt}. Standard Lagrangian type arguments imply the existence of a multiplier $\lambda > 0$ such that the solution satisfies $\mu^\ell = (\omega^\ell \bar{C}^{1-\sigma}/\lambda)^{\frac{1}{\sigma}}$ for all $\ell \in \mathbb{L}$ and $\sum_{\ell\in\mathbb{L}} \mu^\ell = 1$. Combining these conditions to eliminate $\bar{C}^{1-\sigma}/\lambda$ gives (54). For later reference, let $a = 0$ if $\sigma \neq 1$ and $a = 1$ otherwise and use (11) to write the objective function in (53) as

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell \bar{C}) = a\left(u(\bar{C}) + \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell)\right) + (1-\sigma)\, u(\bar{C}) \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^\ell). \tag{A.6}$$

Using (A.6), the maximum value (53) can be expressed as

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\text{opt}} \bar{C}) = \begin{cases} (1-\sigma) m(\omega) u(\bar{C}) & \sigma \neq 1 \\ m(\omega) + u(\bar{C}) & \sigma = 1 \end{cases} \tag{A.7}$$

where $m(\omega) := \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\text{opt}}(\omega))$. Note that $(1-\sigma)m(\omega) > 0$ whenever $\sigma \neq 1$.

(ii) Let $\tilde{\mu} = (\tilde{\mu}^\ell)_{\ell\in\mathbb{L}} \in \Delta_+$ be arbitrary. Defining $\tilde{\omega}^\ell := (\tilde{\mu}^\ell)^\sigma (\sum_{k\in\mathbb{L}} (\tilde{\mu}^k)^\sigma)^{-1}$ for each $\ell \in \mathbb{L}$, one verifies directly that $\tilde{\mu} = \mu^{\text{opt}}(\tilde{\omega})$ solves (53) under this weighting scheme. □

A.6. *Proof of Theorem 1*

Let a weighting scheme $\omega \in \Delta_+$ be arbitrary but fixed and $\mu^{\text{opt}}(\omega)$ be the unique solution to (53). Denote the efficient solution (52) to the APP (51) by $\bar{\mathbf{A}}^{\text{eff}} = (\bar{C}^{\text{eff}}_t, \Gamma^{\text{eff}}_t)_{t\geq 0} \in \bar{\mathbb{A}}$ and define $\mathbf{A} = (\mathbf{C}_t, \Gamma^{\text{eff}}_t)_{t\geq 0} \in \mathbb{A}$ where $\mathbf{C}_t = (C^\ell_t)_{\ell\in\mathbb{L}} := \mu^{\text{opt}}(\omega)\bar{C}^{\text{eff}}_t$ as in the theorem. To establish that $\mathbf{A}$ is ω-optimal, i.e., solves (48), let $\mathbf{A}' = (\mathbf{C}'_t, \Gamma'_t)_{t\geq 0} \in \mathbb{A}$ be any other feasible allocation where $\mathbf{C}'_t = (C^{\ell'}_t)_{\ell\in\mathbb{L}}$, $t \geq 0$. We have to show that

$$V((\mathbf{C}'_t)_{t\geq 0}; \omega) \leq V((\mathbf{C}_t)_{t\geq 0}; \omega). \tag{A.8}$$

Define aggregate consumption $(\bar{C}'_t)_{t\geq 0}$ induced by $(\mathbf{C}'_t)_{t\geq 0}$ as $\bar{C}'_t := \sum_{\ell\in\mathbb{L}} C^{\ell'}_t$, $t \geq 0$. Then, $(\bar{C}'_t, \Gamma'_t)_{t\geq 0} \in \bar{\mathbb{A}}$ and, since $(\bar{C}^{\text{eff}}_t, \Gamma^{\text{eff}}_t)_{t\geq 0}$ solves the APP (51),

$$\sum_{t=0}^{\infty} \beta^t u(\bar{C}'_t) \leq \sum_{t=0}^{\infty} \beta^t u(\bar{C}^{\text{eff}}_t). \tag{A.9}$$

Let $a = 1$ if $\sigma = 1$ and $a = 0$ otherwise. By (53) and (A.7), we have for all $t \geq 0$

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(C^{\ell'}_t) \leq \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\text{opt}}(\omega)\bar{C}'_t) = a\left(m(\omega) + u(\bar{C}'_t)\right) + (1-\sigma)m(\omega)u(\bar{C}'_t) \tag{A.10}$$

and

$$\sum_{\ell\in\mathbb{L}} \omega^\ell u(C^\ell_t) = \sum_{\ell\in\mathbb{L}} \omega^\ell u(\mu^{\ell,\text{opt}}(\omega)\bar{C}^{\text{eff}}_t) = a\Big(m(\omega) + u(\bar{C}^{\text{eff}}_t)\Big) + (1-\sigma)m(\omega)u(\bar{C}^{\text{eff}}_t). \tag{A.11}$$

Equations (A.10) and (A.11) being true for all $t \geq 0$ and (A.9) then give

$$V((\mathbf{C}'_t)_{t\geq 0}; \omega) = \sum_{t=0}^{\infty} \beta^t \sum_{\ell\in\mathbb{L}} \omega^\ell u(C^{\ell'}_t)$$

$$
\begin{aligned}
&\le a\left(\frac{m(\omega)}{1-\beta}+\sum_{t=0}^{\infty}\beta^t u(\bar{C}'_t)\right)+(1-\sigma)m(\omega)\sum_{t=0}^{\infty}\beta^t u(\bar{C}'_t)\\
&\le a\left(\frac{m(\omega)}{1-\beta}+\sum_{t=0}^{\infty}\beta^t u(\bar{C}^{\mathrm{eff}}_t)\right)+(1-\sigma)m(\omega)\sum_{t=0}^{\infty}\beta^t u(\bar{C}^{\mathrm{eff}}_t)\\
&=\sum_{t=0}^{\infty}\beta^t\sum_{\ell\in\mathbb{L}}\omega^\ell u(C^\ell_t)=V((\mathbf{C}_t)_{t\ge 0};\omega)
\end{aligned}
$$

This proves (A.8) and the claim. □

A.7. Computing the efficient solution (52)

We adopt a standard Langrangian approach also used in GHKT to characterize the solution (52). For brevity, use (43) setting $\mathbf{E}^\ell_t := (E^\ell_{i,t})_{i\in\mathbb{I}}$, $D^\ell_t := D^\ell(S_{1,t}+S_{2,t})$ and adopt the notational convention from footnote 16 of treating $X^\ell_{i,t}$ as a dummy argument of F_i if $i\notin\mathbb{I}_x$. Define Lagrangen multipliers $\lambda_t := (\lambda_{0,t}, ((\lambda^\ell_{i,t})_{i\in\mathbb{I}_0}, \lambda^\ell_{N,t})_{\ell\in\mathbb{L}}, \lambda_{K,t}, \lambda_{S_1,t}, \lambda_{S_2,t})$ for each $t\ge 0$ and $\mu=(\mu_i)_{i\in\mathbb{I}_x}$ and the Lagrangian function

$$
\mathscr{L}\Big(\big(\bar{C}_t, K_{t+1}, \mathbf{K}_t, \mathbf{N}_t, \mathbf{E}_t, \mathbf{X}_t, \mathbf{S}_t,\big)_{t\ge 0}, (\lambda_t)_{t\ge 0}, \mu\Big) := \sum_{t=0}^{\infty}\beta^t u(\bar{C}_t) \tag{A.12a}
$$

$$
+\sum_{t=0}^{\infty}\lambda_{0,t}\Big(\sum_{\ell\in\mathbb{L}}(1-D^\ell(S_t))Q^\ell_{0,t}F_0\big(K^\ell_{0,t}, N^\ell_{0,t}, \mathbf{E}^\ell_t\big)-C_t-K_{t+1}-\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}c_i X^\ell_{i,t}\Big) \tag{A.12b}
$$

$$
+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}}\lambda^\ell_{i,t}\Big(Q^\ell_{i,t}F_i(K^\ell_{i,t}, N^\ell_{i,t}, X^\ell_{i,t})-E^\ell_{i,t}\Big) \tag{A.12c}
$$

$$
+\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}\lambda^\ell_{N,t}\Big(N^{\ell,s}_t-\sum_{i\in\mathbb{I}_0}N^\ell_{i,t}\Big)+\sum_{t=0}^{\infty}\lambda_{K,t}\Big(K_t-\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_0}K^\ell_{i,t}\Big) \tag{A.12d}
$$

$$
+\sum_{t=0}^{\infty}\lambda_{S_1,t}\Big(S_{1,t}-S_{1,t-1}-\phi_L\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\zeta_i X^\ell_{i,t}\Big) \tag{A.12e}
$$

$$
+\sum_{t=0}^{\infty}\lambda_{S_2,t}\Big(S_{2,t}-(1-\phi)S_{2,t-1}-(1-\phi_L)\phi_0\sum_{\ell\in\mathbb{L}}\sum_{i\in\mathbb{I}_x}\zeta_i X^\ell_{i,t}\Big) \tag{A.12f}
$$

$$
+\sum_{i\in\mathbb{I}_x}\mu_i\left(R_{i,0}-\sum_{t=0}^{\infty}\sum_{\ell\in\mathbb{L}}X^\ell_{i,t}\right). \tag{A.12g}
$$

Standard arguments imply that any solution $A=(\bar{C}_t, K_{t+1}, \mathbf{K}_t, \mathbf{N}_t, \mathbf{E}_t, \mathbf{X}_t, \mathbf{S}_t)_{t\ge 0}$ to (51) has to satisfy the first order and complementary slackness conditions. After eliminating as many Lagrangian variables as possible, these conditions can be summarized for all $\ell\in\mathbb{L}$, $i\in\mathbb{I}$, and $t\ge 0$ (suppressing quantifiers when convenient) as:

$$
\lambda_{0,t}=\beta^t u'(\bar{C}_t)=\lambda_{K,t+1} \tag{A.13a}
$$

$$\lambda_{K,t} = \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_K F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_K F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) \tag{A.13b}$$

$$\lambda_{N,t}^\ell = \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_N F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_N F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) \tag{A.13c}$$

$$\lambda_{i,t}^\ell = \lambda_{0,t}(1-D_t^\ell)Q_{0,t}^\ell \partial_{E_i} F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big) \tag{A.13d}$$

$$\mu_i = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \lambda_{0,t}c_i - \zeta_i\big(\phi_L\lambda_{S_1,t} + (1-\phi_L)\phi_0\lambda_{S_2,t}\big) \tag{A.13e}$$

$$\lambda_{S_1,t} = \lambda_{0,t}\sum_{\ell\in\mathbb{L}} D^{\ell'}(S_t)Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) + \lambda_{S_1,t+1} \tag{A.13f}$$

$$\lambda_{S_2,t} = \lambda_{0,t}\sum_{\ell\in\mathbb{L}} D^{\ell'}(S_t)Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) + (1-\phi)\lambda_{S_2,t+1}. \tag{A.13g}$$

Note that $\lambda_{0,t}$ can be interpreted as a shadow price of time t consumption. By the same token, the time t shadow price of energy of type $i \in \mathbb{I}$ produced in region $\ell \in \mathbb{L}$ measured in time t consumption goods can be defined as

$$\hat{p}_{i,t}^\ell := \frac{\lambda_{i,t}^\ell}{\lambda_{0,t}} = (1-D_t^\ell)Q_{0,t}^\ell \partial_{E_i} F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell\big). \tag{A.14}$$

Combining (A.13a) and (A.13b) gives the familiar Euler equation

$$u'(\bar{C}_{t-1}) = \beta u'(\bar{C}_t)(1-D^\ell(S_t))Q_{0,t}^\ell \partial_K F_0\big(K_{0,t}^\ell, N_{0,t}^\ell, (E_{i,t}^\ell)_{i\in\mathbb{I}}\big). \tag{A.15}$$

As the l.h.s in (A.13e) is independent of ℓ and t, we obtain using (A.14)

$$\hat{p}_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \zeta_i\Lambda_t = \hat{p}_{i,t}^{\ell'} Q_{i,t}^{\ell'} \partial_X F_i(K_{i,t}^{\ell'}, N_{i,t}^{\ell'}, X_{i,t}^{\ell'}) - \zeta_i\Lambda_t =: \hat{v}_{i,t} \tag{A.16}$$

for all $\ell, \ell' \in \mathbb{L}$ and $t \geq 0$ and

$$\hat{v}_{i,t} - c_i = \frac{\beta u'(\bar{C}_{t+1})}{u'(\bar{C}_t)}\Big(\hat{v}_{i,t+1} - c_i\Big) \tag{A.17}$$

for all $t \geq 0$. Essentially, (A.16) ensures intratemporally efficient allocation and (A.17) intertemporally efficient extraction of resources.

Assuming that $\lim_{n\to\infty}\beta^{n+1}\lambda_{S_1,t+n} = \lim_{n\to\infty}((1-\phi)\beta)^{n+1}\lambda_{S_2,t+n} = 0$, equations (A.13f) and (A.13g) can be solved forward to obtain

$$\frac{\lambda_{S_1,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty}\beta^n\frac{u'(C_{t+n})}{u'(C_t)}\sum_{\ell\in\mathbb{L}} D^{\ell'}(S_{t+n})Q_{0,t+n}^\ell F_0(K_{0,t+n}^\ell, N_{0,t+n}^\ell, \mathbf{E}_{t+n}^\ell) \tag{A.18a}$$

$$\frac{\lambda_{S_2,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty}\beta^n\frac{u'(C_{t+n})}{u'(C_t)}(1-\phi)^n\sum_{\ell\in\mathbb{L}} D^{\ell'}(S_{t+n})Q_{0,t+n}^\ell F_0(K_{0,t+n}^\ell, N_{0,t+n}^\ell, \mathbf{E}_{t+n}^\ell). \tag{A.18b}$$

Finally, define for $t \geq 0$

$$\Lambda_t := \phi_L\frac{\lambda_{S_1,t}}{\lambda_{0,t}} + (1-\phi_L)\phi_0\frac{\lambda_{S_2,t}}{\lambda_{0,t}}. \tag{A.19}$$

Then, using (A.18a) and (A.18b) in (A.19) gives precisely the condition (56).

A.8. *Proof of Theorem 2*

Combining (64) with the conditions derived in Section 2, one can show directly that the aggregate equilibrium allocation solves the same equations as the efficient solution which were derived in Section 3.5 resp. A.7. This proves the claim. □

A.9. Proof of Theorem 3

The assertion follows directly from Proposition 1 and the transfer policy (67). □

A.10. Proof of Theorem 4

Let $\ell \in \mathbb{L}$ be arbitrary and $(\bar{C}_t^{\text{eff}})_{t\geq 0}$ and $(\bar{C}_t^{LF})_{t\geq 0}$ be the aggregate consumption sequences along the efficient and laissez faire allocation, respectively. By Lemma 3(ii) and Assumption 2, utility of region ℓ along the LF allocation is $U((\mu_\ell^{\text{LF}}\bar{C}_t^{LF})_{t\geq 0}) = a + b\,U((\bar{C}_t^{LF})_{t\geq 0})$ where a and $b > 0$ are constants that depend only on μ^{LF}. Further, by construction and Lemma 3(ii), utility of region ℓ along the allocation $\mathbf{A}^*(\tau^{\text{eff}}, \theta^{\text{LF}})$ is $U((\mu_\ell^{\text{LF}}\bar{C}_t^{\text{eff}})_{t\geq 0}) = a + b\,U((\bar{C}_t^{\text{eff}})_{t\geq 0})$. Thus, $U((\mu_\ell^{\text{LF}}\bar{C}_t^{LF})_{t\geq 0}) < U((\mu_\ell^{\text{LF}}\bar{C}_t^{\text{eff}})_{t\geq 0})$ if and only if $U((\bar{C}_t^{LF})_{t\geq 0}) < U((\bar{C}_t^{\text{eff}})_{t\geq 0})$ which follows directly from the optimality of the efficient allocation (52). □

A.11. Proofs for extensions in Section 6

Establishing the following results requires mostly straightforward modifications of the Lagrangean approach used in Section A.7. The following sections highlight how the arguments change depending on the respective modification. Furthermore, introducing damages in the utility function requires a slight modification of the arguments to prove Theorem 1 which are also explained.

A.11.1. Climate change affecting utility

First, we show that Theorem 1 still holds under the alternative specification (70). Using the same notation as in the proof of Theorem 1, denote the efficient solution (52) by $\bar{\mathbf{A}}^{\text{eff}}$ defining efficient aggregate consumption $(\bar{C}_t^{\text{eff}})_{t\geq 0}$ and climate states $(S_t^{\text{eff}})_{t\geq 0}$. Let $\mu^{\text{opt}}(\omega)$ be the unique solution to (53) which is independent of the additive term $v(S_t)$ in (70) and thus the same as before. Let $\mathbf{A}' \in \mathbb{A}$ be any other feasible allocation defining disaggregated consumption $(\mathbf{C}_t')_{t\geq 0}$ with $\mathbf{C}_t' = (C_t^{\ell'})_{\ell\in\mathbb{L}}$, climate states $(S_t')_{t\geq 0}$, and aggregate consumption $(\bar{C}_t')_{t\geq 0}$ where $\bar{C}_t' := \sum_{\ell\in\mathbb{L}} C_t^{\ell'}$. Clearly,

$$\sum_{\ell\in\mathbb{L}} \omega^\ell \Big(\log(C_t^{\ell'}) - v(S_t')\Big) \leq \sum_{\ell\in\mathbb{L}} \omega^\ell \Big(\log(\mu^{\ell,\text{opt}}(\omega)\bar{C}_t') - v(S_t')\Big). \tag{A.20}$$

Exploiting the logarithmic structure in (70) and (A.20) gives the desired result

$$\begin{aligned}
&V((\mathbf{C'}_t, S_t')_{t\geq 0}; \omega) \\
&= \sum_{t=0}^{\infty} \beta^t \sum_{\ell\in\mathbb{L}} \omega^\ell \Big(\log(C_t^{\ell'}) - v(S_t')\Big)x \\
&\leq \sum_{t=0}^{\infty} \beta^t \sum_{\ell\in\mathbb{L}} \omega^\ell \Big(\log(\mu^{\ell,\text{opt}}(\omega)\bar{C}_t') - v(S_t')\Big) \\
&= \sum_{t=0}^{\infty} \beta^t \left(\sum_{\ell\in\mathbb{L}} \omega^\ell \log(\mu^{\ell,\text{opt}}(\omega)) + \log(\bar{C}_t') - v(S_t')\right) \\
&\leq \sum_{t=0}^{\infty} \beta^t \left(\sum_{\ell\in\mathbb{L}} \omega^\ell \log(\mu^{\ell,\text{opt}}(\omega)) + \log(\bar{C}_t^{\text{eff}}) - v(S_t^{\text{eff}})\right)
\end{aligned}$$

$$= \sum_{t=0}^{\infty} \beta^t \sum_{\ell \in \mathbb{L}} \omega^\ell \Big(\log(\mu^{\ell,\mathrm{opt}}(\omega) \bar{C}_t^{\mathrm{eff}}) - v(S_t^{\mathrm{eff}}) \Big) = V((\mu^{\mathrm{opt}}(\omega)\bar{C}_t, S_t^{\mathrm{eff}})_{t \geq 0}; \omega).$$

To compute the efficient solution, replace utility in the objective function in (A.12a) by (70). The first order conditions (A.13f) and (A.13g) now change to

$$\lambda_{S_1,t} = \beta^t v'(S_t) + \lambda_{0,t} \sum_{\ell \in \mathbb{L}} D^{\ell'}(S_t) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) + \lambda_{S_1,t+1}$$

$$\lambda_{S_2,t} = \beta^t v'(S_t) + \lambda_{0,t} \sum_{\ell \in \mathbb{L}} D^{\ell'}(S_t) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell) + (1-\phi)\lambda_{S_2,t+1}.$$

Proceeding exactly as in the proof from Section A.7, these conditions can be solved forward to obtain the additional term (71) in the social cost of Carbon.

A.11.2. Alternative climate model

Under the alternative specification (72) there is no longer a distinction between permanent and non-permanent CO_2 such that the terms $S_{1,t} + S_{2,t}$ in the Lagrangean function (A.12) are now treated as a single variable S_t which evolves according to (72). The climate constraints (A.12e) and (A.12f) are replaced by the single expression

$$\sum_{t=0}^{\infty} \lambda_{S,t} \Big(S_t - \sum_{\tau=0}^{\infty} \delta_\tau \sum_{\ell \in \mathbb{L}} \sum_{i \in \mathbb{I}_x} \zeta_i X_{i,t-\tau}^\ell \Big). \tag{A.21}$$

This new form affects the partial derivative (A.13e) w.r.t. resource $X_{i,t}^\ell$ which now reads

$$\mu_i = \lambda_{i,t}^\ell Q_{i,t}^\ell \partial_X F_i(K_{i,t}^\ell, N_{i,t}^\ell, X_{i,t}^\ell) - \lambda_{0,t} c_i - \zeta_i \sum_{\tau=0}^{\infty} \delta_\tau \lambda_{S,t+\tau}. \tag{A.22}$$

Furthermore, conditions (A.13f) and (A.13g) are now replaced by the single condition

$$\lambda_{S,t} = \lambda_{0,t} \sum_{\ell \in \mathbb{L}} D^{\ell'}(S_t) Q_{0,t}^\ell F_0(K_{0,t}^\ell, N_{0,t}^\ell, \mathbf{E}_t^\ell). \tag{A.23}$$

Substituting (A.23) into (A.22) and adopting the same arguments as in Section A.7, one verifies that the SCC are now given by (73) while all other results hold unchanged.

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